Prof. J. Sachs
May 26, 2016
FIRST DRAFT
COMMENTS WELCOME
PLEASE QUOTE ONLY
WITH PERMISSION

The Best of Times, the Worst of Times:
Macroeconomics of Robotics

Introduction

There are two opposing narratives of the "robot revolution," by which I mean the rising productivity and falling costs of smart ICT-enabled systems, including robots, artificial intelligence, the Internet of Things, remote monitoring and sensing, and other ICT-based systems. In the positive narrative, highly productive robots do the work of humans, thereby raising output, productivity, leisure, and wellbeing. In the negative narrative, robots eliminate jobs, raising unemployment while lowering real wages and wellbeing. Not only are both narratives coherent; they may occur simultaneously, with richer households boosted by robots while poorer households are immiserized. This brief note clarifies these opposing outcomes.

At the core of robot economics is a technological shift towards more capital-intensive production as robots (and other ICT-based systems) substitute for labor. The result is that output, productivity, and profits rise while wages decline. A combination of rising productivity with falling wages is unusual in modern economic history. Productivity and wages have tended to rise together. Indeed, one of the stylized facts of long-term growth has been the stability of the labor share of income, $s = wL/Y$, where s is the real wage, Y is real output, and L is labor input. Since s may be rearranged as $s = w/(Y/L)$ we see clearly that the stability of s means that wages W and productivity Y/L move in parallel.

In recent years the labor share of income has been in decline in many high-income economies, though both the causes and magnitude of the decline are much debated. I would suggest that part of the decline is a manifestation of the robot revolution. Part or even most of the decline in the labor share might be unrelated to robots; one study argues that the recent decline is due to the rise in the share of housing services in GDP.

If robots indeed cause a rise in productivity and profits with a fall in wages, the macroeconomic implications would be quite different from past productivity increases. The traditional optimism that productivity gains broadly improve living standards would have to be significantly qualified. In a life-cycle perspective, young people mostly own labor while older people mostly own financial wealth. Thus, by pushing up profits while depressing wages, the robot revolution would tend to favor the old generation relative to the young generation and all future generations

(whose main endowment will be their labor income). Moreover, since the young are net savers while the old tend to be net dis-savers, a shift of income towards the old and away from the young would tend to lower the national saving rate.

The net welfare effects would be complicated. The capital-owning older generation alive at the time of the robot revolution would certainly benefit. Matters would be more complicated for the young. On the one hand, their wages decline. On the other hand, the rate of return on saving goes up. The net welfare effect can be positive or negative, depending on the household's time rate of discount (among other factors). With a high enough rate of pure time discount, that is, a low weight on future consumption, the adverse wage effect dominates the positive effect of a higher return on saving, so that young workers suffer a decline in lifetime wellbeing.

Since the robot revolution raises national output, the real-income gains of the old wealth holders is greater than the wage losses of the young workers. In principle, the older generation can therefore compensate the young workers in order to keep both the old and young generations better off than before the robot revolution. This kind of transfer can happen in two ways. First, older people might increase their wn voluntary intra-familial bequests to their children, either through higher transfers while they are still alive (*inter vivos* transfers) or higher bequests upon their death. Alternatively, the government might tax some of the capital windfall of the old generation to transfer income to the young workers.

Intra-family transfers from the old to the young would be more likely to occur in richer households since bequests are a "luxury" good (that is, the share of bequests in household income rises with income). As a result, richer households are more likely than poorer households to buffer the losses of their children through bequests. The implication is that the robot revolution is likely to increase the inequality of income and wellbeing by making the rich richer and the poor poorer.

A Simple Analytical Framework

There are many ways to model the labor-market consequences of robots. I suggest the following simple and illustrative approach. Assume that there are two production processes: (1) a traditional assembly line that uses labor L and machines M; and (2) robots R that use only capital. Assume that assembly line output in period t is $Q_A(t)$ is produced with a Cobb-Douglas production function $Q_A(t) = \tau L(t)^s M(t)^{(1-s)}$, where τ is the fixed level of total factor productivity (TFP) and M is machines. The output of the robot sector is $Q_R(t) = \theta(t)R$, where $\theta(t)$ is the TFP of robots in period t.   I assume that $\theta > 1$, meaning that an investment of 1 unit of output in robots leads to output that is greater than 1 in the next period.

Total output Q(t) is the sum of $Q_A(t) + Q_R(t)$:

(1) $Q(t) = \tau L(t)^s M(t)^{(1-s)} + \theta(t)R(t)$

The total capital stock K in any period is allocated between M and R:

(2) $K(t) = M(t) + R(t)$

Capital is allocated competitively to maximize profits, with equilibrium at the point where the marginal products of M(t) and R(t) are equal. I will assume throughout that the baseline robot productivity is sufficiently high to justify at least some deployment of robots along side the traditional machines.[1] Thus profit maximization entails:

(3) $\partial Q/\partial R = \theta = \partial Q/\partial M = MPK$

Condition (3) in turn implies $M(t) = [\tau(1-s)/\theta(t)]^{(1/s)} L(t)$. Upon substituting in (1) we find that total output may be written as a linear function of L and K as follows, where w is the competitive wage (equal to the MPL) and r is the competitive return on capital (equal to the MPK):

(4) $Q = wL + rK$ with $r = \theta(t) > 1$ and $w(t) = \tau^{(1/s)} s[(1-s)/\theta(t)]^{[(1-s)/s]}$

Notice that as $\theta(t)$ rises, the return to capital rises proportionately while the wage *declines* with an elasticity equal to $-(1-s)/s$. Note also that the higher is the share of capital in the assembly-line production, the larger is the percentage decline in the wage following a given percentage rise in robot productivity.

The underlying economic mechanism by which higher robot productivity drives down the wage is straightforward. When $\theta(t)$ rises, capital shifts from assembly-line machinery M towards robots R. Assembly-line workers now work with less machinery M and so end up with lower productivity.   The labor demand schedule of a profit maximizing firm with machinery M(t) is found by equating the Wage with MPL and then solving for labor L: $L^d(t) = w^{[-1/(1-s)]}(\tau s)^{[-1/(1-s)]}M(t)$. Therefore, labor demand at any given wage falls as M declines. To maintain full employment with a given supply of labor, the wage must fall by $-(1-s)/s$ percent for each percent rise in $\theta(t)$.

Incorporating Robots into an OLG Framework

It is straightforward to incorporate robots into an overlapping generations (OLG) framework. Generations are identified by the period of their birth. Each generation lives for two periods, supplies labor L when young and consumes capital income when old. The young generation at time t consumes $C^Y(t)$, saves $S(t) = w(t)L - C^Y(t)$ and accumulates capital out of saving $K(t+1) = S(t)$. Capital lasts for one period (or

---

[1] I assume that θ is high enough to justify the use of robots, which specifically requires: $\theta \geq \tau(1-a)(L/M)^a$

generation). When old, households of generation t consume $C^O(t+1)$ equal to their capital income $r(t+1)K(t+1)$. Total national income in period t is $Y(t)$ equals $w(t)L + r(t)K(t)$, as in (4). The level of labor input of each generation L is assumed to be fixed and is normalized to $L = 1$.

Thus, the market equilibrium is determined by (5) – (9):

    (5) $Y(t) = w(t) + r(t)K(t)$
    (6) $r(t) = \theta(t)$
    (7) $w(t) = \tau\, s(1-s)^{[(1-s)/s]}\, \theta(t)^{[-(1-s)/s]}$
    (8) $K(t+1) = S(t) = w(t) - C^Y(t)$
    (9) $C^O(t+1) = r(t+1)K(t+1)$

A member of generation t allocates consumption across time to $C^Y(t)$ and $C^O(t+1)$ by maximizing lifetime utility U subject to the lifetime budget constraint:

    (10)   max $U[C^Y(t),C^O(t+1)]$ subject to $w(t) = C^Y(t) + C^O(t+1)/r(t+1)$

For simplicity of illustration, I assume that

    (11) $U[C^Y(t),C^O(t)] = constant + \ln[C^Y(t)] + \delta * \ln[C^O(t+1)]$

Utility maximization yields:

    (12) $C^Y(t) = 1/(1+\delta)\, w(t)$ and $C^O(t+1) = [r(t+1)/(1+\delta)]w(t)$

Substituting (12) into the utility function, we find the indirect utility function:

    (13) $U(t) = constant + 2*\ln(w(t)) + \delta * \ln(r(t+1))$

We can see from (13) the welfare effect of a permanent rise in robot productivity $\theta(t)$ starting at time t. Because a rise of $\theta(t)$ lowers $w(t)$ and raises $r(t+1)$ the overall effect on $U(t)$ may be positive or negative depending on parameter values. If the future is discounted heavily enough, that is, if δ is low enough, the rise in $\theta(t)$ is welfare reducing because the negative consequences of the fall in $w(t)$ are not offset by the rise in $r(t+1)$.

General Equilibrium Effects of the Robot Revolution

Using the model (5) – (9) and (11) we can examine the general equilibrium effects of a permanent rise in θ beginning in period t+1. For simulation purposes, I will suppose that the increase in $\theta(t)$ is a step function occurring in period 5 and anticipated in period 4. Before the productivity shock, the economy is in a stationary equilibrium with all values constant during periods 1-3.

The generation born in period 4 (one period before the robot revolution) receives an unambiguous boost of utility, that is, $U(4) > U(3)$, since $r(5)$ rises while $w(4)$ remains unchanged. Thus, the old generation has higher returns to saving but no loss of wage income in the period before the robot revolution. According to (13), lifetime utility of generation 4 is necessarily increased.

As of period 5, output will rise. The wage $w(5)$ declines relative to $w(4)$, while the capital income rises by more than the wage declines. That is:

$w(5) + r(5)K(5) > w(4) + r(4)K(4)$, so that $r(5)K(5)-r(4)K(4) > -[w(5)-w(4)]$.

Saving by the young generation in period 5 falls because of the fall in w. The welfare effect on generation 5 is ambiguous, because $w(5)$ falls while $r(6)$ rises, so that the net effect on $U(5)$ may be positive or negative compared with initial steady state. If δ is low enough, then $U(5)$ will be lower than the initial utility $U(1) = U(2) = U(3)$.

In Figure 1, $\theta(t) = 4$ for $t < 5$ and $= 6$ for $t \geq 5$. The welfare effect of the robot revolution is unambiguously positive for generation 4 as explained and negative for generations 5 and after given the parameters used in Figure 1: $\delta = 0.5$; $s = 0.5$; $\tau = 5$; $L = 1$.

/Figure 1/

Figure 1 therefore depicts the much-feared narrative of "robots taking away jobs" and leaving all future generations poorer than without the robots. It is indeed counter-intuitive that a positive productivity shock can have such pronounced adverse effects in a competitive equilibrium. The outcome is not inefficient per se, since generation 4 is better off than in the baseline. Yet the productivity shock rather straightforwardly immiserizes all generation 5 and after, and even leads to an absolute decline of GDP starting in period 6 because of a steep fall in national saving that begins in period 5.

It is possible to reverse the decline in future wellbeing by transferring some part of the capital windfall earned by generation 4 to the young in generation 5 and then continuing such transfers from the old to the young in each succeeding period after period 5. This possibility is illustrated in Figure 2, where I introduce a compensating government transfer from the old to young in each time period as well as the lifetime utility of each generation. The transfer is in effect paid for by a tax on capital income (one can think of this as lowering the net-of-tax return on saving for young households). Comparing Figure 1 and Figure 2 we see a truly fundamental lesson: *the robot revolution can potentially benefit all future generations but only if inter-generational transfers from the old to the young are part of a social contract between overlapping generations.*

/Figure 2/

The Bequest Motive and Generational Wellbeing

There are two ways to achieve the needed inter-generational transfers shown in Figure 2. One is through a government-sponsored tax-and-transfer policy as depicted in Figure 2, in which the government taxes the capital income of the old generation and transfers the proceeds to the young generation with a balanced budget. This is a pay-as-you-go *reverse* social security system, in which today's old generation makes transfers to young workers. The reversal in direction of intergenerational transfers from the usual social security is needed to offset the shift of national income towards capital and away from labor. (Presumably, for a small rise in θ(t), it may be enough to reduce the size of current flows from young to old rather than reversing the direction of the flows).

The other way this intergenerational transfer system can work is through voluntary bequests or *inter vivos* transfers within each family. To illustrate this possibility, we must introduce bequests into the model. An easy way to do this is to assume that parents care about the wellbeing of their children as measured by the income of their children inclusive of the bequests. Thus, let B(t+1) be the bequest in period t+1 that is given by the old of generation t to their children of generation t+1. The total income of the young workers in period t+1 is therefore w(t+1) + B(t+1). We will assume generation 4 anticipates as of period 4 the coming decline in their children's earnings w(5).

We now modify the utility function to include bequests:

(14) $U(t) = U[C^Y(t), C^O(t+1), w(t+1)+B(t+1)]$ subject to

$w(t)+B(t) = C^Y(t)+C^O(t+1)/r +B(t+1)/r$

Note that households of generation t both receive bequests B(t) from their parents of generation t-1 and leave bequests B(t+1) to the children of generation t+1.

In specifying the bequest motive, we take into account the empirical pattern that bequests are made mainly by richer households. A very simple functional form of the utility function with this property is the following:

(15) $U(t) = \ln(C^Y(t)) + \ln(C^O(t+1))/\delta +$
$\alpha(w(t+1)+B(t+1)) - \beta[w(t+1)+B(t+1)]^2$

According to (14), the utility of generation t is logarithmic in own consumption and quadratic in the bequest motive. As a result, households in generation t with a low income w(t) + B(t) will leave zero bequests B(t+1) = 0 to their own children. Households that inherit a large bequest B(t) will tend to leave a large bequest to their children B(t+1).

Optimizing U(t) according to the budget constraint in (13) gives a bequest function for generation t as illustrated in Figure 3 for the parameters w = 1.563 , α = 0.2, β = 0.02, δ = 0.5 , θ = 4. The bequest made by generation t in period t+1, B(t+1), is a function of the bequest received by generation t when young, B(t). Figure 3 is therefore a map from the bequest level received from parents to the bequest given to children. Households that receive large bequests when young also make large bequests when old. Households that receive no bequests when young make no bequests when old.

By superimposing a 45-degree line in Figure 3, we show in Figure 4 that there are three bequest equilibria for an inter-generational family dynasty: a zero-bequest equilibrium in which young households in every generation receive no bequests from their parents and leave no bequests to their children; and intermediate level B** that is unstable; a locally stable high-bequest equilibrium B* in which every generation receives B* from their parents and leaves B* to their children. If a family line starts generation t with a bequest of B** or less, the family bequest will fall to zero over time and the family will end in the zero-bequest equilibrium. If a family enters generation t with a bequest greater than B**, then the bequest will grow over time and the family will reach a high-bequest equilibrium at B*.

In a long-term equilibrium, there will be poor households stuck at B(t) = 0 for all t, and rich households with B(t) = B* at all time. The robot revolution will hurt the future family line with B(t) = 0 but benefit the family line with B(t) = B*. In short, the robot revolution will make the rich richer and the poor poorer.

It might seem at first glance that if a household receives a bequest B* and later makes an equal bequest B* that it is not benefited on net (other than a warm-glow effect, perhaps, of receiving and giving bequests). Yet this is not correct. Receiving B* when young and paying B* when old signifies net wealth in present value terms:

PV = (B* - B*/r) = [(r-1)/r]B* > 0 with r > 1.

Consider now a family line at the high-bequest equilibrium, B(t) = B*. Let's again examine the effect of a rise in robot productivity starting in period 5 and anticipated as of period 4. Generation 4 enjoys a capital windfall and also recognizes that their children will soon experience a decline in wages, w(5) < w(4). The anticipated fall in their children's wage will trigger a rise in their voluntary bequest B(5) > B* in order to help compensate for the decline in w(5). Thus generation 4 voluntarily leaves part of its windfall to their children. Generation 5 in turn leaves some of its own income to their children in generation 6, and so forth. Every generation now benefits from the rise in robot productivity and leaves a bequest that is higher than the original equilibrium level B* to their own children. The adverse consequences to future generations of the robot shock are eliminated through voluntary intra-familial bequests (or *inter vivos* transfers).

Now consider the households initially at the zero-bequest equilibrium, with B(1) = ... = B(4) = 0. If the productivity rise is large enough, generation 4 will leave a small bequest B(5) to their children, but B(5) will likely be much less than B** so that the family will not make a long-term transition to the high-bequest equilibrium. Indeed, with the parameters used in Figure 4, generation 4 leaves a small bequest to their children in generation 5, but generation 5 does not leave bequests to their children. The family dynasty quickly reverts to the zero-bequest equilibrium as soon as generation 6. All generations 5 and after of this poor family suffer a decline in lifetime wellbeing relative to the baseline utility of generations 1-3 before the robot revolution.

It is clear, then, that if rich and poor households live side by side, the rich households will make intra-family transfers to that leave all future generations better off while the poor households do not do so and suffer an absolute decline of wellbeing. This is illustrated in Figure 5, which shows the utility of rich and poor households over time. The robot shock has therefore not only worsened the wage level and left many households poorer, but it has also widened the income inequality across the households of any generation. Finally, note that the future generations of poor families too could benefit from the robot revolution but only through *fiscal policy* (i.e. taxing the older generation and distributing the income to the young) rather than voluntary intra-family bequests.
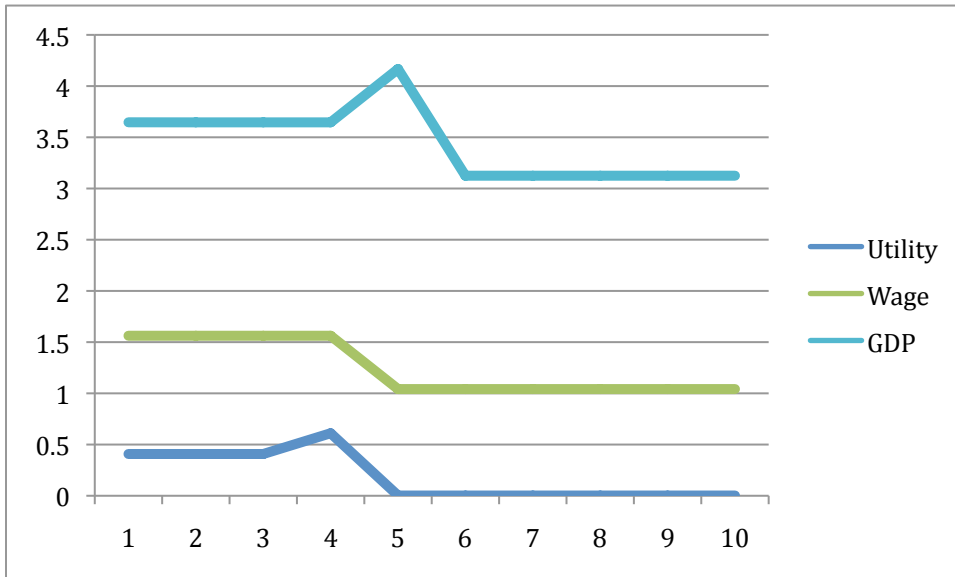
/Figure 5/

Conclusions and Extensions

The robot revolution is likely to raise capital income, lower labor income, and redistribute earnings from the young to the old. As we have seen, this can result in impoverishment of the young and of future generations, matched by a one-time windfall of the older generation that is alive when the robot revolution occurs. Alternatively, all generations can benefit if inter-generational transfers from old to young are made in the current and future periods. These transfers may occur as voluntary bequests within families or through a reverse social-security scheme in which wealth holders are taxed in order to pay for transfers to young workers.

This is a very simple illustrative model. There are obviously many ways to make this work more quantitative and realistic. The production functions themselves could be specified with more precision. Realistic intergenerational dynamics could be added, involving multiple overlapping generations and numerical calculations of generational accounts.

We could add in different skill levels of workers, with high-skilled workers gaining from the robot revolution and low-skilled workers losing. We might think of R as ICT-supported *human capital* rather than as robots per se. In this case, skilled parents may fund the higher education of their children, who become high-paid

skilled workers, while unskilled parents are unable or unwilling to fund the education of their children, who then grow up to become low-paid low-skilled workers. It is indeed worrisome in this context that the intergenerational correlation of educational attainment in the US is higher than in most other OECD countries, probably because of the high tuition costs of US universities. In this case, the robot revolution may indeed lead to a widening inequality of income between households with higher human capital accumulation and those with lower human capital accumulation.

Figure 1. Effect of a Rise in Robot Productivity in Period 5

Notes: Generation 4 enjoys a rise in lifetime wellbeing, while all Generations 5 and later experience a decline. GDP rises in Period 5 and then declines thereafter due to the fall in saving. The Wage w declines beginning in Period 5.

Figure 2. Rise in Robot Productivity Combined With Transfers From Old to Young
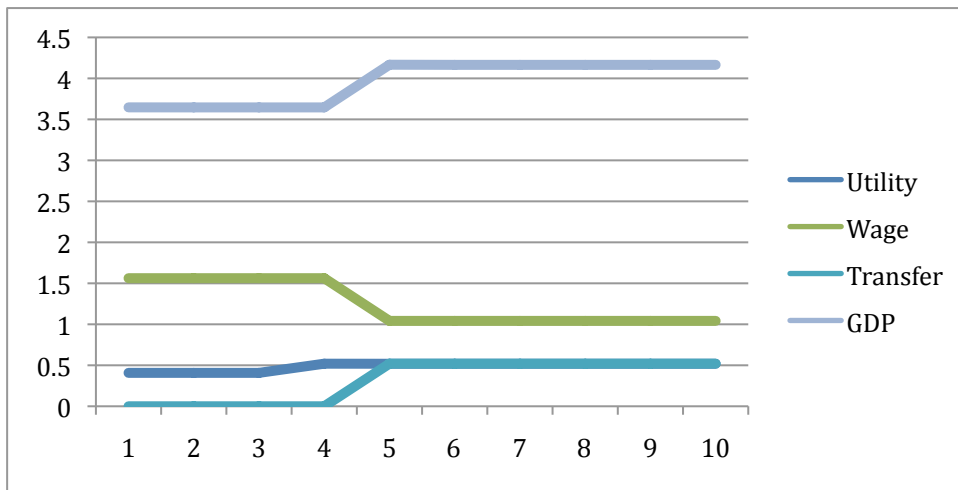
Notes: Beginning in Period 5, the government taxes the capital income of the old generation and transfers the revenues to the young generation. Utility is thereby increased for all generations 4 and after and GDP rises in Period 5 and after.

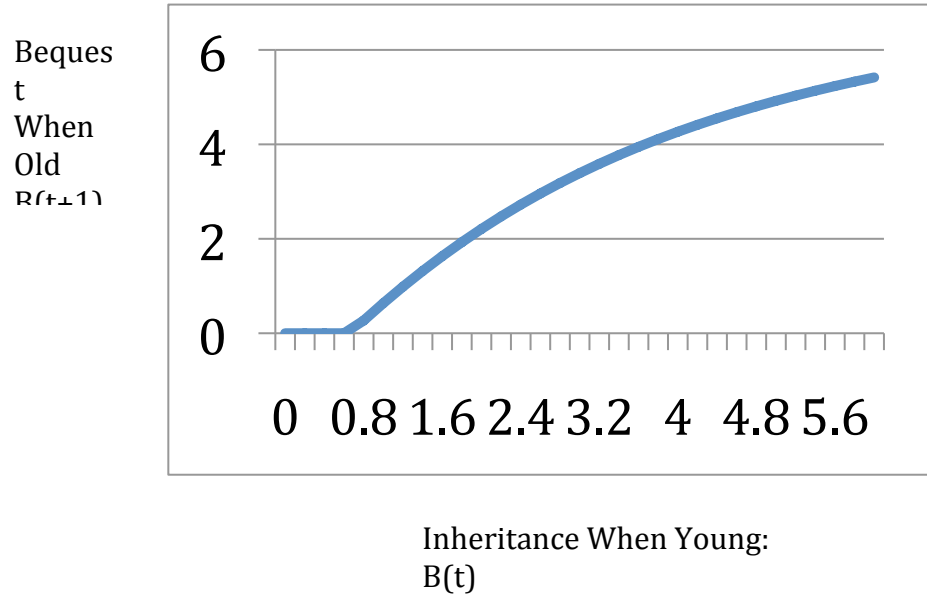Figure 3. The Relationship Between Inheritance B(t) and Bequests B(t+1)

Bequest
When
Old
B(t+1)

Inheritance When Young:
B(t)

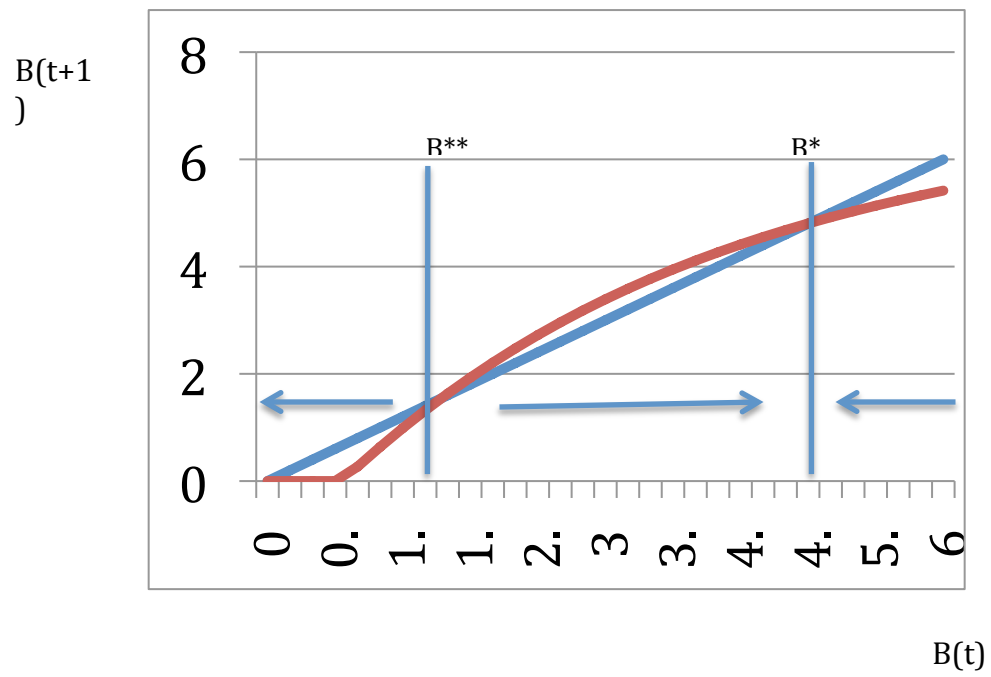Figure 4. Bequest Dynamics: Two Stable Stationary Points, 0 and B*
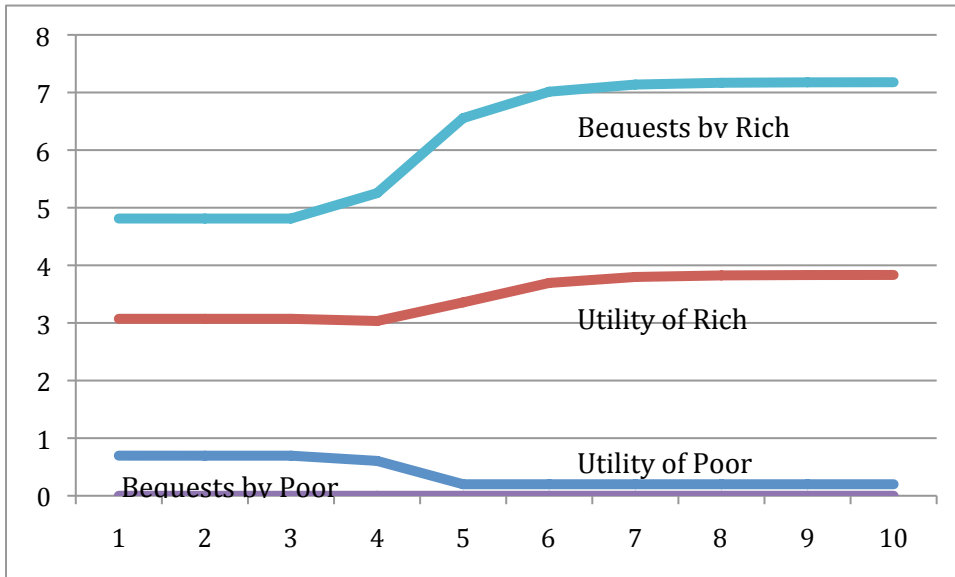
Figure 5. Rising Inequality Following Robot Revolution

Note: Rich Households Boost their Bequests In Order to Share Windfall across generations. Poor Households have Zero Bequests. Rich households Enjoy a rise in lifetime utility while poor households experience a decline.