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ABSTRACT

Do robots raise or lower economic well-being? On the one hand, they raise output and bring more goods and services into reach. On the other hand, they eliminate jobs, shift investments away from machines that complement labor, lower wages, and immiserize workers who cannot compete. The net effect of these offsetting forces is unclear. This paper seeks to clarify how economic outcomes, positive or negative, depend both on specific parameters of the economy and public policy. We find that a rise in robotic productivity is more likely to lower the welfare of young workers and future generations when the saving rate is low, automatable and non-automatable goods are more substitutable in consumption, and when traditional capital is a more important complement to labor. In some parameterizations the relationship of utility to robotic productivity follows a "noisy U" as large innovations are long-run welfare improving even though small innovations are immiserizing. Policies that redistribute income across generations can ensure that a rise in robotic productivity benefits all generations.

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1 Introduction

The word robot comes from the Czech word ‘robota’, meaning forced labor. Ever since the term’s invention in Karl Čapek’s 1920 dystopian science fiction masterpiece R.U.R, it has been associated with ambivalence about the power of automation. The play begins with the general manager of Rossum’s Universal Robots discussing the potential of his assembled beings to raise living standards. He predicts that his robot laborers will lower the prices of goods to zero, ending toil and poverty for all time. This plan hits a small snag when the robots decide to overthrow their masters and destroy all humans. But was the manager’s economic forecast even correct in the first place?

This paper investigates the implications of capital investments, in the form of robots, which allow for production without labor. Our key finding is that an increase in robotic productivity will temporarily raise output, but, by lowering the demand for labor, can lower wages and consumption in the long run. In what we term a *paradox of robotic productivity*, innovations that increase the productivity of robotic investments can, after a generation, lower robotic and total output, and lower the well-being (lifetime utility) of all future generations. The mechanism for this immiserization is decreased wages of the workers with whom the robots compete. We find this immiserization is most likely when the future is heavily discounted, goods produced by robots are close substitutes for goods created by human labor, and when traditional capital is a more important factor in non-robotic production (so that the reduction of traditional capital has a larger adverse impact on wages). In our richest setting, increases in robotic productivity lower well-being until a threshold is reached. After reaching the threshold, the economy may grow indefinitely.

The fact that a rise in robotics productivity may immiserize future generations is paradoxical. After all, higher productivity enables society to produce more output for the same level of inputs. If the market response to robotic innovations does not lead to a positive result, this suggests that there may be an role for government intervention. We show this intuition to be correct. Immiserization may be overcome through redistributive policies of the state.

The paper proceeds as follows. A brief literature review puts current concerns about automation in a historic context and surveys the research on robots and growth. Section 3 introduces a basic overlapping generations setting in which the generational impact of robots can be considered. Section 4 investigates the one-sector version of the model, and section 5 analytically considers the two-sector version. Section 6 gives a numerical analysis of the two-sector model. Section 7 concludes.

2 Literature Review

Even before the birth of modern science fiction, academics and ordinary people have been concerned about the potential downsides of technological growth.[1] The English Luddites of the late 18th and early 19th centuries famously organized raids and riots against the industrial machines they felt were taking their jobs. In the second half of the 19th century, Marx (1867) bemoaned the fact that under capitalism "all methods for raising the social productivity of labour are put into effect at the cost of the individual worker". In the first half of the 20th century Keynes (1933) cautioned against overreaction to "technological unemployment", which, while painful for displaced workers, was merely a "temporary phase of maladjustment". Similarly, Schumpeter championed the "creative destruction" of capitalism, in which older ways of doing work are, not without pain, superseded by advances in technology as new types of more productive work are created.

In the economic prosperity of the post-war era, the views of technological optimists generally held sway. However, recent wage stagnation and growing inequality across the developed world have led economists to take another hard look at technological growth. Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011), and Autor and Dorn (2013) trace recent declines in employment and wages of middle skilled workers to the development of smart machines. Margo (2013) points to similar *labor polarization* during the early stages of America's industrial revolution. Goos, Manning, and Salomons (2010) offer additional supporting evidence for Europe. Sachs and Kotlikoff (2013) present a model in which robots immiserize future generations, a precursor of the models studied in this paper. However, Mishel, Shierholz, and Schmitt (2013) argue that 'robots' can't be 'blamed' for post-1970's U.S. job polarization given the observed timing of changes in relative wages and employment. A literature inspired by Nelson and Phelps (1966) hypothesizes that inequality may be driven by skilled workers more easily adapting to technological change, but generally predicts only transitory increases in inequality.

A potential implication of our model is a decline, over time, in labor's share of national income. U.S. national accounts record a stable percent share of national income going to labor during the 1980's and 1990's. But starting in the 2000's labor's share has dropped significantly. Benedict and Osborne (2013) try to quantify prospective human redundancy arguing that over 47 percent of current jobs will likely be automated in the next two decades. Hemous and Olson (2014) calibrate a model in which capital can substitute for low-skilled labor while complementing high-skilled labor to explain trends in the labor share of income and inequality.

The lessons of our model are also related to the endogenous growth literature. In Rebelo's (1991) AK model, sustained per capita output growth occurs so long as there are no decreasing returns to scale in production. This model complemented Romer (1990) which included open ended growth driven by endogenous technological development in the tradition of learning by doing proposed by

[1] This section draws on Benzell *et. al.* (2015).

Arrow (1962).

There are several models incorporating intergenerational transfers that find the possibility for welfare improving transfers. Two papers that with mechanisms more similar to this one are Sachs and Kotlikoff (2013) and Benzell *et. al.* (2015). These papers also posit that technological changes may immiserize future generations through the mechanism of reduced wages.

3 The Model Framework

The essential quality of robots, as we define them, is that they allow for output without labor. To produce a unit of output from robotic technology, entrepreneurs need only make a capital investment. Innovation in robotic production can therefore change labor's share of national income. In a model with an infinitely lived representative consumer, this is unlikely to have major effects. However, if those earning labor and capital income have different propensities to consume, then a change in labor's share of income can have important effects on saving and investment. We attempt to capture this effect in the simplest possible setting.

The setup is an overlapping generations (OLG) model with two cohorts. This allows for labor's share of income to have a dynamic effect and straightforward generational welfare analysis. Agents consume, work, and save while young, and consume when old.

Households

All individuals live for two periods, working, saving and consuming while young, and just consuming while old. Workers in this economy maximize a lifetime utility function of the form

$$U_t = \phi u(\vec{c}_{1,t}) + (1-\phi)u(\vec{c}_{2,t+1}), \tag{1}$$

where $\vec{c}_{1,t}$ and $\vec{c}_{2,t+1}$ are vectors of goods consumed by a household in the first and second periods of life, and $u(\cdot)$ is a within-period homothetic utility function. Henceforth, we will assume the within-period utility is logarithmic, $u(\vec{c}_t) = ln(v(\vec{c}_t))$, where v is Cobb-Douglas with constant returns to scale. There is no leisure.

A generation maximizes U_t subject to its lifetime budget constraint, which in general may include government taxes and transfers.

$$w_t L_t + G_t = \vec{p}_t \vec{c}_{1,t} + \frac{\vec{p}_{t+1}\vec{c}_{2,t+1}}{1 + [r_{t+1}(1-\tau_t)]}, \tag{2}$$

where $\vec{p}_t$ is a vector of prices, w_t is the wage, G_t is the size of government grants to the young, $1 + r_t$ the interest rate, and τ_t is the capital income tax rate. For convenience, define the net income of the young as the sum of their labor income and any government transfer, and the net interest rate of the old as net of the government capital income tax. So,

$$w_t^N = w_t L_t + G_t, \tag{3}$$

and

$$r_t^N = r_t(1-\tau_t). \tag{4}$$

Utility maximization leads to the well-known result that saving, S_t, equals a fixed fraction $(1-\phi)$ of youth income,

$$S_t = (1-\phi)(w_t^N). \tag{5}$$

Households allocate savings with perfect foresight between available types of physical assets to maximize returns.

4 The One-Sector Model

In this model framework the performance of labor markets is strongly linked to the extent goods produced with human effort are replaceable by those that robots create. When the outputs of robots are close substitutes for production by humans and machinery, an increase in robotic productivity is likely to reduce demand for labor. A fall in labor demand may trigger further declines in wages, saving, and economic well-being. However, to the extent that workers produce outputs that are imperfect substitutes of the outputs of robots, workers will experience a rise in demand for their products, and this can result in a virtuous circle of rising wages, savings, and production.

First we consider the one-sector version of the model in which the traditional and robotic production technologies produce the same good.

Supply in the One-Sector Model

In the one-sector model, there are two types of firms. Time t production of the consumption and investment good with the traditional output technology, $X_{m,t}$, follows

$$X_{m,t} = D_{X,t} M_{X,t}^{\epsilon} L_{X,t}^{1-\epsilon}, \tag{6}$$

where $M_{X,t}$ is the amount of machines rented by these firms, $L_{X,t}$ is the amount of labor hired, ϵ a Cobb Douglas parameter, and $D_{X,t}$ a total factor productivity term. Production by robotic firms follow

$$X_{r,t} = \Theta_t R_t, \tag{7}$$

where $X_{r,t}$ is the output of these firms, R_t is the amount of robots rented by these firms, and Θ_t is the robotic productivity.

Factor demands for robots, machines, and labor reflect

$$\max_{M_{X,t}L_{X,t}} X_{m,t}(M_{X,t}, L_{X,t}) - w_t L_{X,t} - m_t M_{X,t} \tag{8}$$

and

$$\max_{R_t} X_{r,t}(R_t) - \rho_t R_t, \tag{9}$$

where m_t is the rental rate for machines, and ρ_t is the rental rate for robots.

These yield the first order conditions

$$w_t = (1-\epsilon) D_{X,t} M_{X,t}^{\epsilon} L_{X,t}^{-\epsilon}, \tag{10}$$

$$m_t = \epsilon D_{X,t} M_{X,t}^{\epsilon-1} L_{X,t}^{1-\epsilon}, \tag{11}$$

and

$$\rho_t = \Theta_t. \tag{12}$$

Households in the One-Sector Model

Utility is logarithmic in consumption of the one good.

$$u(x_t) = ln(x_t), \tag{13}$$

Household demands for consumption and investment satisfy

$$x_{1,t} = \phi w_t^N \tag{14}$$

and

$$x_{2,t} = (1 + r_t^N) K_t, \tag{15}$$

where K_t is capital of any type owned by the old.

Equilibrium in the One-Sector Model

The total output of the economy is the sum of the outputs of the two types of firms,

$$X_t = X_{m,t} + X_{r,t}. \tag{16}$$

The one-sector model is in equilibrium when the market for goods clears,

$$X_t = x_{1,t} + x_{2,t} + S_t, \tag{17}$$

the labor market clears,

$$L_{X,t} = L_t, \tag{18}$$

the government is balancing its budget,

$$G_t = r_t \tau_t K_t, \tag{19}$$

and the market for investments clears,

$$S_t = K_{t+1} = M_{X,t+1} + R_{X,t+1}, \tag{20}$$

as capital depreciates fully each period.

Finally, investment seeks the highest return in the subsequent period with perfect foresight, and here we are only interested in the case where robots are productive enough to be used, so investment must equalize the rate of return of both forms of capital. Therefore,

$$1 + r_t = m_t = \rho_t = \Theta_t. \tag{21}$$

One-Sector Equilibrium Analysis

Consider the case where $D_{X,t} = L_t = 1$ in all periods.

Combining first order equations yields

$$w_t = (1-\epsilon)\frac{\epsilon}{\Theta_t}^{\frac{\epsilon}{1-\epsilon}}. \tag{22}$$

Note that a rise in robot productivity reduces the wage. The reason is that higher Θ shifts investment from machines into robots. This lowers the capital-labor ratio in X_m firms, decreasing the marginal productivity of workers.

We can write the indirect utility function in terms of Θ_t and Θ_{t+1}. Ignoring constant terms, and assuming no transfers ($G_t = \tau_t = 0$) we have

$$U_t = lnw_t + (1-\phi)ln(1+r_{t+1}), \tag{23}$$

or,

$$U_t = \frac{-\epsilon}{1-\epsilon}ln\Theta_t + (1-\phi)ln\Theta_{t+1}. \tag{24}$$

Notice that robot productivity has two opposing effects on lifetime utility. A rise of Θ_t lowers the wage while a rise of Θ_{t+1} raises the returns on saving. The negative wage effect tends to dominate the saving effect the larger is the capital share of income (ϵ) in the machine using firms is large, because this measures the importance of machines in complementing the labor or workers. Immiserization is also more likely when the discount rate ϕ is higher, because a high ϕ means that the utility value of higher returns to saving is low.

Consider a one-step permanent rise of Θ at time T. That is for $t < T$, $\Theta_t = \Theta^L$ and for $t \geq T$, $\Theta_t = \Theta^H > \Theta^L$. The lifetime utility across generations is therefore as follows:

for $t < T-1$

$$U_t = \frac{-\epsilon}{1-\epsilon}ln\Theta^L + (1-\phi)ln\Theta^L, \tag{25}$$

when $t = T-1$

$$U_t = \frac{-\epsilon}{1-\epsilon}ln\Theta^L + (1-\phi)ln\Theta^H, \tag{26}$$

and if $t > T-1$

$$U_t = \frac{-\epsilon}{1-\epsilon} ln\Theta^H + (1-\phi) ln\Theta^H. \quad (27)$$

The rise in robot productivity in period T raises the welfare of generation $T-1$. For that generation, the rise of robot productivity has not yet affected wages. However, the return on saving increases by the rise in robotic productivity in period T. Generation $T-1$, in other words, will enjoy high wages when young and high retirement income due to the surge of robot technology. Generations T and after will not be so lucky. For them, the mixed effects of better robots will be reflected in lower wages and a higher rate of return to saving.

An increase in robotic productivity will induce long-run immiserization as long as[2]

$$\frac{\epsilon}{1-\epsilon} > (1-\phi). \quad (28)$$

If (28) holds, the wage effect dominates and leads to a decline in lifetime utility. Thus, when the parameter values allow for immiserization, only a single generation benefits from the rise of robot productivity, specifically the generation born just before the improvement in robot productivity. That generation benefits from higher returns to saving without incurring the negative shock of lower wages.

Ensuring that all generations benefit from the rise in Θ

Could a managed rise of robots lead to a better long-run outcome? It is clear that markets alone are not sufficient to ensure that a rise of robot productivity raises the well-being of future generations. However, it seems likely that a pure rise in productivity Θ, by pushing out the production possibility frontier, can be made into a rise in lifetime utility for all generations with the right kind of government intervention. To insure a better outcome, the income of the young should be augmented by redistribution from the old.

Here's how to turn the robotics innovation in time T into a rise in well-being for all generations from time T-1 onward.

In every period T and after, the government levies a tax on the capital income of retirees and transfers the proceeds as a grant G_t to the young. Let the government set the grant equal to the decline of the wage caused by the rise of Θ. Let w^H be the market wage associated with Θ^H and w^L be the market wage associated with Θ^L. Then necessarily, $w^L > w^H$. The grant mechanism will function as follows: For $t > T-1$

$$G_t = w_t^L - w_t^H. \quad (29)$$

[2]On the other hand, a reduction in long-run national consumption can only occur if Θ increases above 1. This is because the golden rule (long-run consumption maximizing) level of saving, given constant L and full depreciation is that which brings long-run interest rates equal to 1. In cases where Θ increases from a level below 1 to a level closer to but still below 1, long-run consumption will increase although welfare may decrease.

To pay for this grant, the government levies a capital-income tax at rate τ_t on the old in each period. With saving S_t, pre-tax capital income is given by $\Theta^H S_t$. Therefore, the tax rate should be set such that for $t \geq T$

$$G_t = (\Theta^H - 1)\tau_t K_t. \tag{30}$$

Of course, savers anticipate this capital income tax and plan their inter-temporal spending decisions accordingly. Instead of earning a rate of return Θ^H, savers will earn a net-of-tax rate of return $(1 + r^N_{t+1}) = 1 + (\Theta^H - 1)(1 - \tau_t)$. Because of their logarithmic preferences this change in rate of return does not change their saving behavior. The indirect lifetime utility function can be re-written in terms of youth net-of-transfer income w^N_t and r^N_{t+1}. Since policy fixes the disposable wage at w^L_t we have, ignoring constant terms,

$$U^L_t = ln(w^L_t) + (1 - \phi)ln(1 + r^N_{t+1}). \tag{31}$$

Every generation will be better off when Θ rises to Θ^H, as net of tax lifetime budget constraints must be larger than when Θ^L.

When Θ rises, it is easy to see that X_t rises instantaneously as well. This is because the level of capital is unchanged, but its productivity has increased. Now, consider total output from the perspective of factor income. Since there are no profits, $X_{r,t} = \Theta R_t$ and $X_{m,t} = w_t + \Theta M_t$, we have that $X_t = w_t + \Theta(R_t + M_t) = w_t + \Theta S_{t-1}$. By (5), S_t depends only on the net income of the young w^N_t. The transfer system keeps the disposable wage equal to w^L_t, so saving S_t also remains unchanged when Θ rises. When Θ rises, the overall rise of X_t ensures that $w^H_t + \Theta^H S_t > w^L_t + \Theta^L S_t$. Therefore, $w^H_t - w^L_t + \Theta^H S_t > \Theta^L S_t$. Since $w^L_t - w^H_t$ equals G_t, which is also equal to $(1 + (\Theta^H - 1)\tau_t)S_{t-1}$, we find that $(1 + (\Theta^H - 1)\tau_t)S_{t-1} > \Theta^L S_t$. Hence, $(1 + r^N_{t+1}) = (1 + (\Theta^H - 1)\tau_t) > \Theta^L$.

This reasoning establishes a key result. By taxing the capital of the old, and transferring the proceeds to the young, the government keeps the net income of the young unchanged while the net-of-tax rate of return on saving is higher. Therefore, the rise of robot productivity to Θ^H combined with the fiscal transfer system raises the well-being of all generations compared with the utility when productivity equals Θ^L.

The result is important in light of discussions as to whether robotics will necessarily raise or lower well-being. The answer is that higher productivity is a potential gain for all generations, but only if government undertakes redistributive policies to ensure that indeed all generations benefit. Without such redistribution, it is possible, we have seen, that the robotics innovation improves the well-being of just one generation, while lowering the lifetime well-being of all future generations.

5 The Two-Sector Model

An important critique of the one-sector model is that it takes robotic and labor produced goods as identical. In reality, there are many goods that robots cannot create or might only create with greatly diminished productivity. Examples include many personal services that depend intrinsically on human-to-human interactions, and various kinds of creative activities not reducible to computer algorithms, e.g. in the arts.

To allow for complementaries in consumption between robotic and non-robotic goods, we move to a richer two-sector setting. Here two goods are produced and consumed, but only one is automatable (i.e. eligible for production by robots). The core insights of the one-sector model are maintained, but complex additional dynamics emerge.

Supply in the Two-Sector Model

In the two-sector model, there are three types of firms. The X sector is identical to the one-sector case, so

$$X_{m,t} = D_{X,t} M_{X,t}^{\epsilon} L_{X,t}^{1-\epsilon}, \tag{32}$$

and

$$X_{r,t} = \Theta_t R_t. \tag{33}$$

But in addition there are firms producing the consumption good Y, with technology

$$Y_t = D_Y M_{Y,t}^{\alpha} L_{Y,t}^{1-\alpha}, \tag{34}$$

where Y_t is the output of these firms at time t, $M_{Y,t}$ is the amount of machines rented by these firms, $L_{Y,t}$ is the amount of labor they hire, α is capital's share of output in production of Y, and D_Y is a total factor productivity term.

We will refer to the X sector as the robotic or automatable sector interchangeably. We will refer to the Y sector as the non-robotic, non-automatable, or traditional sector interchangeably.

Factor demands for robots, machines, and labor reflect

$$\max_{M_{X,t} L_{X,t}} X_{m,t}(M_{X,t}, L_{X,t}) - w_t L_{X,t} - m_t M_{X,t}, \tag{35}$$

$$\max_{R_t} X_{r,t}(R_t) - \rho_t R_t, \tag{36}$$

and

$$\max_{M_{Y,t} L_{Y,t}} p_t Y_t(M_{Y,t}, L_{Y,t}) - w_t L_{Y,t} - m_t M_{Y,t}, \tag{37}$$

where p_t is the price of the non-automatable good in terms of the potentially robotic good. All factor inputs must be non-negative.

Assuming that the non-negative input constraint does not bind for any type of firm, first order conditions are

$$w_t = (1 - \epsilon_t) D_{X,t} M_{X,t}^{\epsilon} L_{X,t}^{-\epsilon}, \tag{38}$$

$$w_t = (1-\alpha_t)p_t D_{Y,t} M_{Y,t}^{\alpha} L_{Y,t}^{-\alpha}, \tag{39}$$

$$m_t = \epsilon_t D_{X,t} M_{X,t}^{\epsilon-1} L_{X,t}^{1-\epsilon}, \tag{40}$$

$$m_t = \alpha p_t D_{Y,t} M_{Y,t}^{\alpha-1} L_{Y,t}^{1-\alpha}, \tag{41}$$

and

$$\rho_t = \Theta_t. \tag{42}$$

Households in the Two-Sector Model

Within period utility is logarithmic in the Cobb-Douglas combination of the two types of consumption.

$$u_t(x_t, y_t) = \beta ln(x_t) + (1-\beta) ln(y_t). \tag{43}$$

This specification implies that individuals want to spend constant shares of their consumption on the automatable and non-automatable good.[3]

The household budget constraint is

$$w_t^N = x_{1,t} + p_t y_{1,t} + \frac{x_{2,t+1} + p_{t+1} y_{2,t+1}}{1 + r_{t+1}^N}. \tag{44}$$

Household demands for consumption and investment at time t satisfy

$$x_{1,t} = \beta\phi w_t^N, \tag{45}$$

$$x_{2,t} = (1 + r_t^N)\beta K_t, \tag{46}$$

$$y_{1,t} = \frac{(1-\beta)\phi w_t^N}{p_t}, \tag{47}$$

$$y_{2,t} = \frac{(1-\beta)(1 + r_t^N) K_t}{p_t}, \tag{48}$$

and

$$S_t = (1-\phi) w_t^N, \tag{49}$$

where K_t is capital of any type owned by the old.

[3]This is an important assumption. We do not have a strong intuition about whether technological innovations will shift consumption demand towards or away from goods that are relatively labor intensive. Good arguments can be made for both perspectives. If demand does indeed shift towards robotic goods, then our immiserizing mechanism will be amplified and vice versa.

Equilibrium in the Two-Sector Model

The potentially robotic good is an investment and consumption good, while the non-robotic sector produces only a consumption good. Capital depreciates fully each period. Equilibrium requires

$$X_t = X_{m,t} + X_{r,t}, \tag{50}$$

$$X_t = x_{1,t} + x_{2,t} + S_t, \tag{51}$$

$$Y_t = y_{1,t} + y_{2,t}, \tag{52}$$

$$L_t = L_{X,t} + L_{Y,t}, \tag{53}$$

$$G_t = r_t \tau_t K_t, \tag{54}$$

and

$$S_t = K_{t+1} = M_{X,t+1} + M_{Y,t+1} + R_{X,t+1}. \tag{55}$$

Phases of the Two-Sector Economy

As in the one-sector model, investors allocate capital with perfect foresight to maximize returns. Here this means that the non-negative input constraint sometimes binds. When robotic productivity and capital stocks are low, it is inefficient to invest in robots, and only traditional manufacturing will be done in the automatable sector. When robotic productivity and capital stocks are high, traditional manufacturing is not competitive in the X sector, and only robotic investments are made. Finally, there is a range of values for Θ_t and K_t where both traditional manufacturing and robots are used in the automatable sector. Taking $G_t = \tau_t = 0$, this will occur whenever,

$$K_t < \Theta_t^{\frac{-1}{1-\epsilon}} \left(\left((1-\epsilon) D_{X,t}^{\frac{1}{1-\epsilon}} \epsilon^{\frac{\epsilon}{1-\epsilon}} \right) \left(\frac{\phi\beta}{1-\beta} + \frac{\phi\alpha}{(1-\alpha)(1-\beta)} \right) \right), \tag{56}$$

and

$$K_t > \Theta_t^{\frac{-1}{1-\epsilon}} \left(\phi L_t D_{X,t}^{\frac{1}{1-\epsilon}} \epsilon^{\frac{\epsilon}{1-\epsilon}} [(1-\beta)\alpha + \beta\epsilon] \right). \tag{57}$$

When (57) is violated, no machines or labor are used in the automatable sector. When (56) is violated, the model reduces to the normal two-sector OLG and no robots are used. Note that when $D_{X,t} = 0$ it must be the case that no labor is used in the automatable sector as its productivity must be zero. As $\Theta_t \to 0$, there is never enough capital to make robotic production competitive and only the first case is possible.

The paper proceeds by considering these three cases in turn.

Case 1: No Robots Used

When capital stocks per unit of labor are low, the marginal productivity of capital is high. If capital stocks per unit of labor are low enough, investing all savings in the form of traditional machines will yield a higher interest rate than Θ, the rate of return on robots. In such periods, the interest rate will not be fixed by Θ but will be a function of capital stocks. It must be that $1 + r_t > \Theta_t$. The economy will behave as in the well-known two-sector OLG case.

Over time, the economy will grow or contract towards a steady state level of capital. If the $\Theta = 0$ steady state level of capital is high enough, accumulating capital stocks will eventually cross the threshold level given by (57), and some savings will be invested in the form of robots. One of the following two cases will hold. Exogenous changes in parameters, importantly Θ, may also move the economy into one of the following phases.

As this type of economy is well understood, we will now move on to cases of greater interest.

Case 2: Mixed Production of the Automatable Good

When both (56) and (57) hold, robots and traditional manufacturing compete head to head in the creation of the same products. $R_t, L_{Y,t}$, and $L_{X,t} > 0$. Optimization requires $1 + r_t = \Theta_t$.

Insights from the one-sector model carry over into this case. Assume for now that there are no transfers. Combining first order conditions, the price of the non-automatable good may be written as

$$p_t = \frac{\Theta_t}{\alpha D_{Y,t}} [\frac{M_{t,y}}{L_{t,y}}]^{1-\alpha}. \tag{58}$$

Factor demands also imply

$$\frac{M_{t,y}}{L_{t,y}} = \frac{\alpha(1-\epsilon)}{\epsilon(1-\alpha)} \frac{M_{t,x}}{L_{t,x}}, \tag{59}$$

and

$$\frac{M_{t,x}}{L_{t,x}} = \Big(\frac{\epsilon D_{X,t}}{\Theta_t}\Big)^{\frac{1}{1-\epsilon}}. \tag{60}$$

This allows for the rewriting of prices in terms of Θ_t.

$$p_t = \Theta_t^{\frac{\alpha-\epsilon}{1-\epsilon}} \frac{1}{\alpha D_{Y,t}} \Big(\frac{\alpha(1-\epsilon)}{\epsilon(1-\alpha)}\Big)^{1-\alpha} (\epsilon D_{X,t})^{\frac{1-\alpha}{1-\epsilon}}. \tag{61}$$

Equation (61) demonstrates two important properties of this economy. First, prices do not depend on the level of capital. While the economy uses all three productive processes, capital and labor migrate across sectors keeping prices fixed. Second, Θ_t has an ambiguous effect on the price. When capital intensity in the traditional sector, α, is less than the capital intensity of labor-based production in the robotic sector, an increase in Θ_t lowers prices and vice-versa. This is because the increase in robotic productivity draws capital away from investment in both types of machines, and this will affect output of the more machine intensive sector more. A larger reduction in output requires a change in relative prices. As it is intuitive that capital should be more important in the production of the automatable good, we take this to be the standard case.

To better understand how α influences the impact of a change in robot productivity, consider $\alpha = 0$. Also, take $L_t = 1$ in all periods.

The first order condition for the non-automatable good reduces to

$$p_t = \frac{w_t}{D_{Y,t}}. \tag{62}$$

So the price will be a function of the wage, which can be thought of as being set in the X sector. In the X sector, an increase in robotic productivity will redistribute capital investment from machines to robots. Because this reduces the marginal productivity of labor while leaving the price of X unchanged, wages must decrease.

Taking the limit of the price equation as $\alpha \to 0$ yields

$$p_t = \Theta_t^{\frac{-\epsilon}{1-\epsilon}} \frac{1}{D_{Y,t}} [\frac{(1-\epsilon)}{\epsilon}][\epsilon D_{X,t}]^{\frac{1}{1-\epsilon}}. \tag{63}$$

Prices and wages are both decreasing in Θ.

The wage in the general case will be

$$w_t = p_t(1-\alpha) D_{Y,t} [\frac{M_{t,y}}{L_{t,y}}]^{\alpha}, \tag{64}$$

which can be rewritten as

$$w_t = p_t(1-\alpha) D_{Y,t} [\frac{\alpha(1-\epsilon)}{\epsilon(1-\alpha)}]^{\alpha} [\frac{\epsilon D_{X,t}}{\Theta_t}]^{\frac{\alpha}{1-\epsilon}}. \tag{65}$$

The wage is not a function of capital either. This means that in the period after a change in Θ the economy will jump to its new steady state.

Explicitly,

$$K_{t+1} = C_1 [\frac{1}{\Theta_t}]^{\frac{\epsilon}{1-\epsilon}}, \tag{66}$$

where

$$C_1 = \frac{(1-\phi)(1-\epsilon)}{\epsilon} (\epsilon D_{X,t})^{\frac{1}{1-\epsilon}}.$$

Note that wages and future capital are decreasing in Θ_t.

Plugging wages and interest rates into the utility function yields an indirect utility function in terms of parameters

$$U_{1,t}(\Theta) = \tilde{C} - [\frac{\epsilon + \phi(1-\beta)(\alpha-\epsilon)}{1-\epsilon}] \ln \Theta_t + (1-\phi)[1 - \frac{(1-\beta)(\alpha-\epsilon)}{1-\epsilon}] \ln \Theta_{t+1}, \tag{67}$$

where $\tilde{C}$[4] is a function of parameters other than robotic productivity. The utility of the young is always decreasing in today's robotic productivity, while the effects of robotic productivity in their retirement has ambiguous.

The long-run impact welfare impact of a permanent increase in Θ will be negative if

$$(1-\phi) < \frac{(1-\beta)\alpha}{1-\epsilon} + \frac{\epsilon\beta}{1-\epsilon}. \tag{68}$$

The impact of increased robotic productivity will be positive if the discount factor is low enough. When labor-based production of the robotic good is more capital intensive, robotic productivity changes will tend to be more damaging to welfare as more labor will be forced out of robotic production and into lower marginal product tasks. Similarly, when the capital share of production in the Y sector is small, output of the non-automatable good is more resistant to reallocation of investment, and the threshold for immiserization is higher.

By similar logic as in the one-sector model, a government transfer can turn an increase in robotic productivity into a long-term welfare improvement. Government transfers of the type discussed above will not change the pre-transfer wage, and therefore must increase capital stocks that are linear in post-transfer wage. An increase in capital stocks must increase output. If the transfer is set so as to bring w_t^N after innovation equal to w_t before the innovation, no profits necessitates that the old consume more because total output has increased.

An economy can evolve move from this case to either the no-robot or only robot case in one of two ways. Most simply, if a parameter such as Θ were to change then either equation (56) or (57) may bind. More subtly, if $K_{t+1}(\Theta_t)$ is large or small enough then an economy in the mixed case at (K_t, Θ_t) will immediately jump to one of the other cases. This can lead to permanent cycles in the economy if in the only-robot case the economy contracts.

Case 3: Only Robots Produce the Automatable Good

In the final case of the economy, robotic productivity is so high that no machines or labor are used in the automatable sector. This is intuitive, as when labor is relatively scarce firms should substitute for it as much as they can.

Without transfers, and economy in this case is set on a path towards a permanent growth or temporary contraction similarly to an AK model. The potential for permanent growth arises from the fact that the rise of Θ raises the relative

[4] $\tilde{C} = \phi\beta \ln(\beta\phi) + \phi(1-\beta)\ln([1-\beta]\phi) + (1-\phi)\beta\ln(\beta) + (1-\phi)(1-\beta)\ln(1-\beta) + (1-\beta)(\frac{1-\epsilon}{1-\alpha}\ln(\alpha D_{X,t}) - \ln(\epsilon D_{Y,t}) + (1-\epsilon)\ln(\frac{\epsilon[1-\alpha]}{\alpha[1-\epsilon]})) + \phi\ln(\frac{1-\epsilon}{\epsilon}) + \frac{\phi}{1-\epsilon}\ln(\epsilon D_{X,t})$

price of Y, which can in turn raise the wage, the level of saving, and investment. At the initial price level, a rise in Θ shifts capital to robotic investment, thereby raising the output of X and lowering the output of Y. Yet demand for the traditional good rises because retirees boost their overall demand, of which traditional consumption is a fixed share. The result is an excess demand for the traditional good, requiring a rise in prices to clear the market. As the price rises, so too can wages. The effect on wages will be the net of the increase in price and the decrease in the marginal productivity of labor due to capital flight. If there is an increase in the wage, this causes a rise in national saving and thereby a rise in investment. With more saving there is also more demand for the traditional good, which is limited by the fixed supply of labor. An ongoing cycle of growth will continue despite the fixed input of labor.

Robots will necessarily be utilized, so $1 + r_t = m_t = \Theta_t$. The non-negativity constraint for inputs to machine production of the automatable good binds, so $L_{Y,t} = L_t$ and $M_{X,t} = K_t - R_t$. Assume that there are no government transfers. Then rearranging first order conditions yields

$$w_t L_t = M_t \frac{\Theta_t(1-\alpha)}{\alpha}. \tag{69}$$

Combining the robotic production function with the robotic market clearing condition yields

$$\Theta_t R_t = x_{1,t} + x_{2,t} + K_{t+1}, \tag{70}$$

and substituting household demands gives,

$$\Theta_t[K_t - M_t] = (1-\phi)w_t L_t + \phi\beta w_t L_t + \beta\Theta_t K_t, \tag{71}$$

which may be reduced to

$$M_t = \frac{\alpha(1-\beta)}{1-(1-\alpha)\phi(1-\beta)} K_t, \tag{72}$$

giving a law of motion for capital

$$K_{t+1} = \frac{(1-\beta)(1-\phi)(1-\alpha)}{1-(1-\alpha)\phi(1-\beta)} \Theta_t K_t. \tag{73}$$

Thus, capital evolves linearly across periods. When the term multiplying K_t is less than 1, the economy will contract. When greater than 1, it will grow indefinitely. Note that this term is not dependent on D_Y but it is increasing in robotic productivity. This is because increases in the price of the traditional good guarantee that a stable fraction of the robotics output is devoted to saving for more robots. Increased robotic productivity may lead the world from poverty into permanent growth but increasing traditional productivity will have no effect on growth rates. If total factor productivity in the traditional sector were to increase, its price would drop by precisely the amount to keep the wage constant. The multiplier is also increasing in the saving rate $1-\phi$. Government interventions to increase saving have the potential to move the economy from steady contraction to unconstrained growth.

When the economy is on the contraction side of knife-edge growth, savings and capital will decrease until (57) no longer holds. If the case that the economy moves into then leads to an increase in capital stocks, the economy may remain in an endogenous business cycle of growth and contraction indefinitely. An example is given in the simulations below.

It is easy to see that the knife-edge growth case will have a positive long-run effect on utility. The knife edge growth case is growing precisely because wages are increasing, and the increase in capital stocks (while the interest rate remains constant) indicates that the old have higher incomes as well.

To better understand the difference between the one-sector model and this phase of the two-sector model, consider $\alpha = 0$, that is, no machines are used in producing Y. Then the two production functions are

$$X_t = \Theta_t R_t, \tag{74}$$

$$Y_t = D_{Y,t} L_t, \tag{75}$$

and since L = 1,

$$Y_t = D_{Y,t}. \tag{76}$$

First, consider what happens when the two goods are perfect substitutes as in the one-sector model. The wage w_t is simply $D_{Y,t}$, and the economy immediately reaches a steady state with

$$\overline{R} = (1-\phi) D_{Y,t}, \tag{77}$$

and

$$\overline{X} = \Theta_t (1-\phi) D_{Y,t}. \tag{78}$$

There is no growth. A rise in Θ increases lifetime utility for all generations by raising the return on saving. There is no adverse wage effect, as there is no capital flight to reduce the productivity of labor.

Now consider the very different outcome in the two-sector, only-robots case. The wage w_t now equals $p_t D_{Y,t}$. Saving is $S_t = (1-\phi) p_t D_{Y,t}$. Total demand for X_t is

$$X_t = \phi \beta p_t D_{Y,t} + \Theta_t \beta R_t + (1-\phi) p_t D_{Y,t}. \tag{79}$$

We therefore can find p_t by equating the supply and demand for X_t. Specifically,

$$p_t = \Theta_t (1-\beta) R_t / [\phi \beta D_{Y,t} + (1-\phi) D_{Y,t}]. \tag{80}$$

Using the relationship $R_{t+1} = S_t = (1-\phi) p_t D_{Y,t}$, we find a difference equation in R_t,

$$R_{t+1} = \Theta_t (1-\beta)(1-\phi) R_t / [\phi\beta + (1-\phi)]. \tag{81}$$

In both this and the more general case, a fixed share of robotic output is devoted to investment.

When only robots are used for producing the automatable good, transfers still have the potential to increase long-run welfare. For transfers satisfying

$$-\frac{[1-(1-\beta)\phi]\left(\frac{1-\alpha}{\alpha}\right)+1}{\left(\frac{1-\alpha}{\alpha}\right)[\theta(1-\beta)+\beta T]} < 1-\phi < \frac{[1-(1-\beta)\phi]\left(\frac{1-\alpha}{\alpha}\right)+1}{\left(\frac{1-\alpha}{\alpha}\right)[\theta(1-\beta)+\beta T]}, \tag{82}$$

the economy will converge to a steady state. Otherwise the economy will experience AK growth/contraction. This means that the economy has the potential to be shunted out of contraction by a transfer.

When (82) holds, capital stocks converge to

$$K^{ss} = \frac{(1-\phi)T}{1 - \frac{(1-\phi)(\frac{1-\alpha}{\alpha})}{[(1-(1-\beta)\phi)\left(\frac{1-\alpha}{\alpha}\right)+1]}}. \tag{83}$$

6 Simulating The Two-Sector Model

In figures 1 and 2 we display the path of an economy with parameters given in table 1. These figures demonstrate how a government transfer program can turn a potentially utility-reducing rise in robotic productivity into a welfare improvement for all generations. In this simulation in all periods the model is in case two, where both robots and machines used in the production of the automatable good. From periods zero through four, the economy is in its steady state. In period five Θ increases from 1.25 to 2. Without transfers, this leads to a temporary boom. High savings carried over from period four are combined with the new technology and create high levels of output, most of which accrue to the old due to the decrease in labor's share of income. From period five on, citizens suffer as a result of the increase in productivity. Welfare falls far below the dashed line indicating their utilities had the technological innovation never occurred.

Introducing a transfer changes the outlook for the economy. Capital income taxes, set at rate of about 70 percent, fund a transfer that keeps the net income of the young constant. This keeps capital stocks constant while prices are unchanged. Relatively higher capital stocks outweigh the impact of the tax and increase the net income of the old. Every generation benefits from the combination of technological change and transfers.

Figures 3 and 4 investigate a more complex case. The economy begins in period zero just below the steady state level of capital given initial Θ. For the initial level of Θ, in the steady state robots are too inefficient to be used. The economy is in case one. In this second pair of simulations, we investigate the consequences of robotic productivity innovations occurring every five periods beginning in period five.

First, consider the consequences for the economy without transfers. After the first innovation, the economy moves into case two (mixed production of the automatable good). Welfare hits a local maximum as the old receive large retirement incomes from the interest rate increase. But the increase in robotic investment lowers wages. In the period after the innovation utilities move to their new lower level due to lower wages and capital. In period ten another innovation occurs, but the economy remains in the second case. Another local maximum in welfare follows, before welfare falls even further.

In period fifteen innovators strike again. In the period of the third innovation, there is a third local maximum in the utility of the old. The economy has moved into the third case where only robots are used in production. However, the multiplier on K_t in (73) is less than one, and the economy immediately begins to contract because of low wages. Wages are low because the negative wage effect of losing an opportunity for employment in the automatable sector dominates the positive effect of increases in non-automatable good prices (which are in turn due to their increased relative scarcity).

After a single period capital has dissipated enough that case two binds again. In period sixteen, although capital is scarcer, wages are higher than in the previous period because workers are being used to produce the automatable good again. High wages increase savings and future capital, moving the economy into the third case. Periods where only robots are used have low wages, reducing savings. The economy is on the bad side of knife-edge AK growth. In subsequent periods, capital stocks are low enough that the second case binds. Laborers find work again in the automatable sector and wages increase. Capital stocks and the economy expand, moving the economy back into the third case. These oscillations have important welfare implications as those retired in periods where robots are used and working when case two binds have high wages when young and high retirement incomes when old. Those unlucky to be born in the other period of the business cycle are worse off. Cycles of more than one period are possible, although the economy will not spend more than one period in case two per cycle.

The economy would oscillate indefinitely were it not for a final innovation in period twenty. This moves the economy on to the good side of knife-edge growth. The positive effect on wages of high non-automatable good prices dominates. The economy grows indefinitely with benefits for all future generations. For a wide variety of parameterizations, a noisy U-shaped path of utility as Θ increases will occur. In early periods robotic productivity leads to immiserization, but eventually robots are so super-productive that indefinite growth must kick in.

The path of welfare can be improved through government transfers. Here is displayed one of a large set of transfer schedules that turn the series of innovations into an improvement for all generations over robots being banned. Curiously, the transfer improves welfare by depressing labor's share of income even further in some periods. This is due to greater investment, which requires the output of the more capital intensive X sector and leads to higher future capital income.

Figure 5 shows the long-run impact of a robotic productivity improvement on an economy in case two. Unsurprisingly, when the saving rate is high the increase in robotic technology (and hence interest rates) is more likely beneficial. When capital's share of income in production of the traditional good is higher, robotic innovations are more likely to immiserize by crowding out investment in a more important complement to labor.

7 Conclusion

The rise of the robots is already creating major disruption in labor markets, essentially turning production processes more capital intensive. When robots are close substitutes for production by labor and machinery, the demand for labor is likely to decline, threatening a decline of wages, saving, and economic well-being of current and future generations. We have qualified that intuition, however, in two important ways. First, government redistribution can ensure that a pure productivity improvement raises well-being of all generations. In the example shown in the paper, government taxes the capital owned by retirees and distributing the proceeds to young workers. Second, to the extent that workers produce outputs that are imperfect substitutes of the outputs of robots, workers will experience a rise in demand for their products, and this can result in a virtuous circle of rising wages, savings, and production, producing the open-ended constant growth of an AK model.

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Table 1
Parameters for First Simulation

Model Parameter	Role	Value
ϵ	X Sector Capital Input Share Param.	0.33
α	Y Sector Capital Input Share Param.	0.33
Θ	Robot Productivity	Varies
$1-\phi$	Saving Rate	0.3
L	Labor Supply	1
G	Transfer to Young	Varies
β	X Sector Consumption Share	0.5
K_0	Initial Capital	.104
$D_{X,t}$	TFP in X Sector	1
D_Y	TFP in Y Sector	1

Table 1: This table gives parameter values for the first pair of illustrations of the model.

Table 2
Parameters for Second Simulation

Model Parameter	Role	Value
ϵ	X Sector Capital Input Share Param.	0.33
α	Y Sector Capital Input Share Param.	0.33
Θ	Robot Productivity	Varies
$1-\phi$	Saving Rate	0.3
L	Labor Supply	1
G	Transfer to Young	Varies
β	X Sector Consumption Share	0.5
K_0	Initial Capital	.29
D_X	TFP in X Sector	1
D_Y	TFP in Y Sector	1

Table 2: This table gives parameter values for the second pair of illustrations of the model.

Figure 1
Simulation 1: Welfare

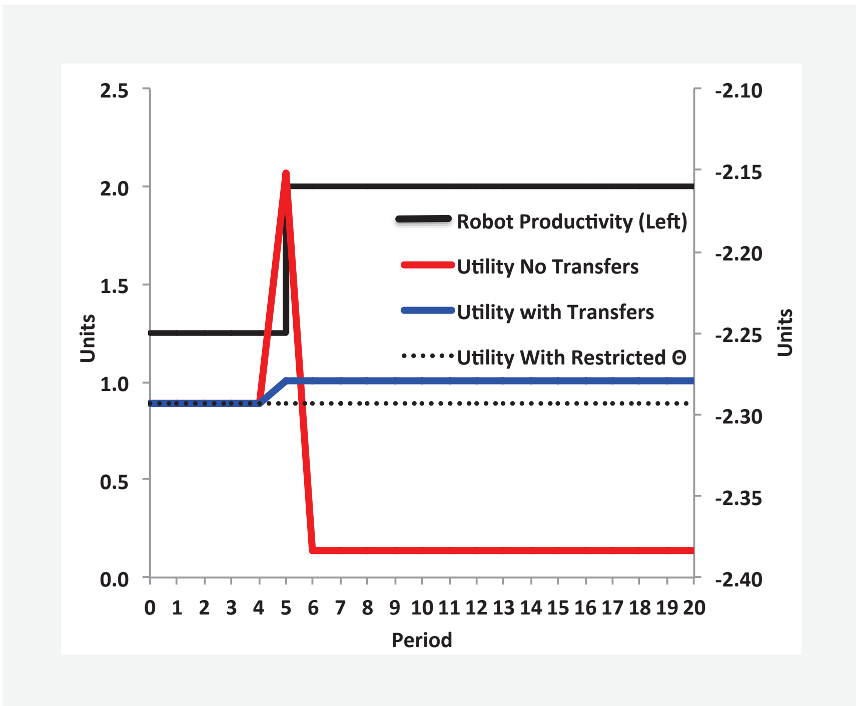

Figure 1: Utility and Θ paths for an economy with and without transfers before and after an increase in Θ. Welfare is lifetime utility of those retired in a period. Parameter values are as in Table 1.

Figure 2
Simulation 1: Other Economic Variables

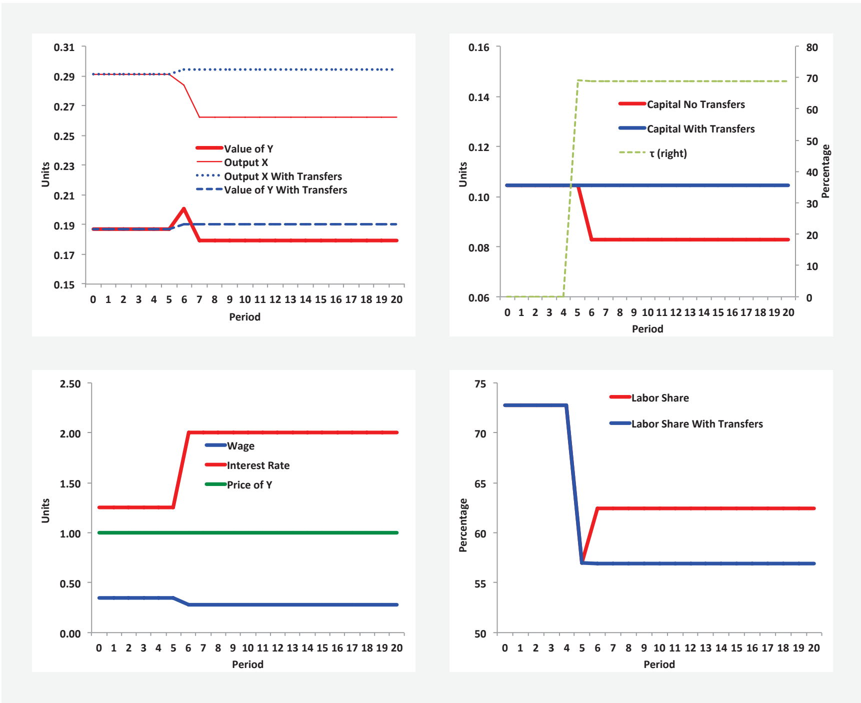

Figure 2: Economic variable paths for an economy with and without transfers before and after an increase in Θ. Wage is before transfers. All prices are identical with and without transfers. Parameter values are as in Table 1

Figure 3
Simulation 2: Welfare

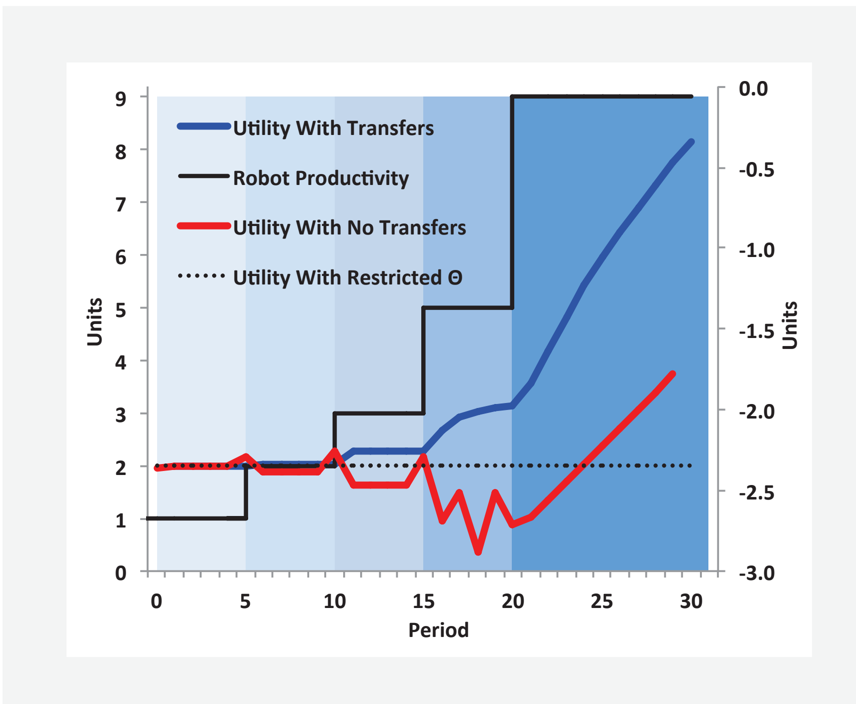

Figure 3: Utility paths for an economy with and without transfers before and after several increases in Θ. Welfare is lifetime utility of those retired in a period. Parameter values are as in Table 2.

Figure 4
Simulation 2: Other Economic Variables

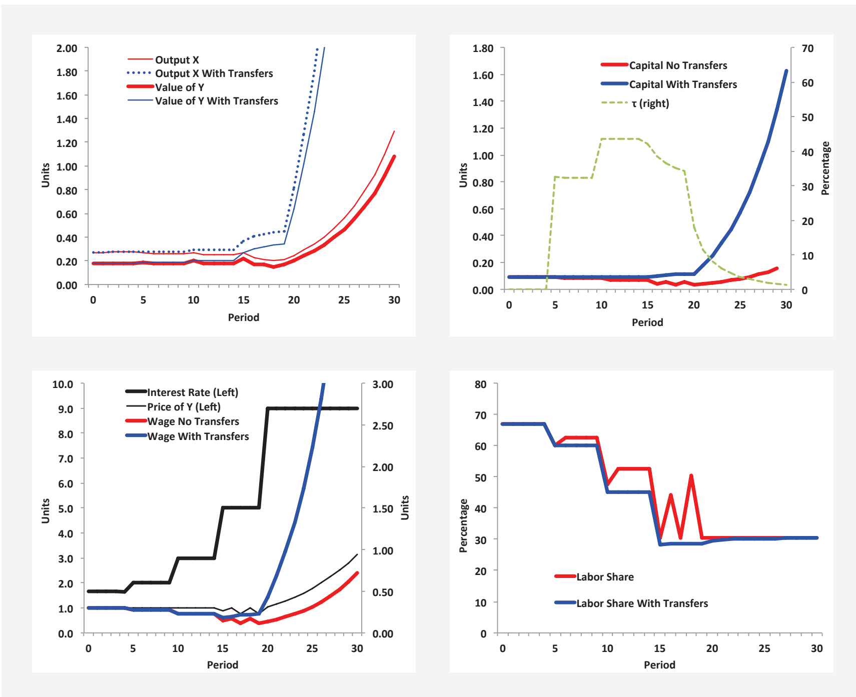

Figure 4: Economic variable paths for an economy with and without transfers before and after several increases in Θ. Parameter values are as in Table 2.

Figure 5
Role of Parameters in Determining the Welfare Impact of $\Delta\Theta$ in the Mixed Case

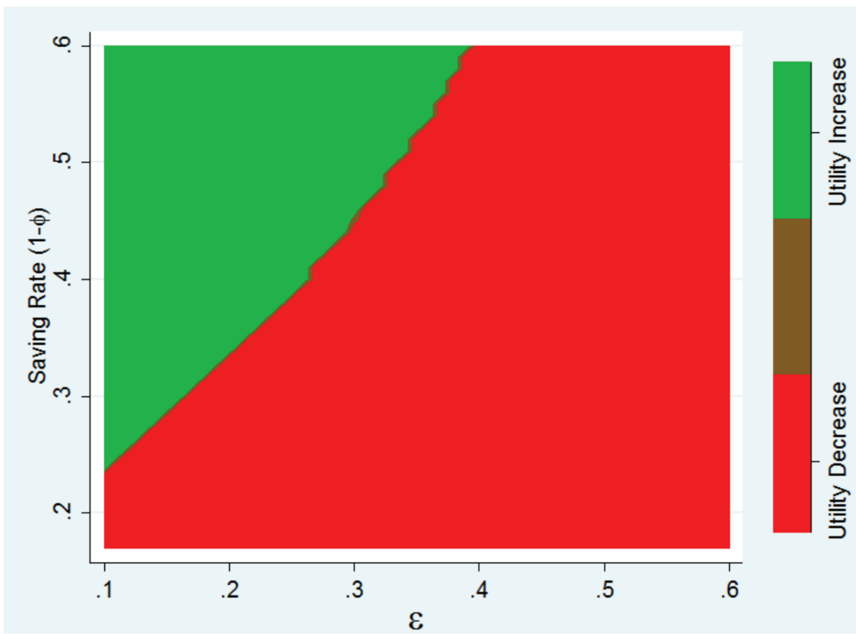

Figure 5: The green zone indicates indicates the range of parameter values such that an increase in robotic productivity has a positive long-run impact on utility; for the red zone the opposite holds. The economy begins in the steady state with $\Theta = 1.25$ and is compared to the steady state with robotic productivity slightly elevated. Parameters not on axes are as in table 1