NBER WORKING PAPER SERIES

THE MCKIBBIN-SACHS GLOBAL MODEL: THEORY AND SPECIFICATION

Warwick J. McKibbin

Jeffrey D. Sachs

Working Paper No. 3100

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1989

The authors would like to thank Peter Boone, Graham Elliott, Nouriel Roubini, and Eric Siegloff for their contributions to the development of the MSG model and Alan Powell and Peter Wilcoxen for comments. The authors also thank the Brookings Institution, Ford Foundation, and National Science Foundation for financial support. This paper is part of NBER's research program in International Studies. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research nor the other institutions mentioned above.

NBER Working Paper #3100
September 1989

THE MCKIBBIN-SACHS GLOBAL MODEL: THEORY AND SPECIFICATION

ABSTRACT

This paper presents the theoretical underpinnings of the MSG2 simulation model of the world economy. The MSG2 model is a dynamic general equilibrium model of the world economy which pays particular attention to the relation between stocks and flows and intertemporal constraints. The formation of expectations also plays an important role in the model. In the version presented here the world is divided into the U.S., Japan, Germany, the rest of the EMS, and the rest of the OECD, non-oil developing countries and OPEC.

Warwick J. McKibbin
The Brookings Institution
1775 Massachusetts Ave, N.W.
Washington, DC 20036

Jeffrey D. Sachs
Department of Economics
Harvard University
Cambridge, MA 02138

I. Introduction

Major imbalances in the world economy, large swings in asset prices and the renewed focus on international policy coordination have lead economists and policymakers to look to large-scale macroeconometric models for some guidance in understanding the macroeconomic inter-relationships between countries. In the early 1980's, the existing global models proved disappointing in explaining the large movements in asset prices and trade balances at that time. The MSG2 modelling project began at the National Bureau of Economic Research in 1985 in an attempt to use insights from modern macroeconomic theory and game theory to understand these key developments in the world economy.

This paper provides a detailed derivation and description of the MSG2 model of the world economy. Section II provides an overview of the methodology behind the modelling project. A detailed description of the model is provided in Section III. In section IV, issues of model calibration and solution are discussed. The properties of the model a explored in depth in McKibbin and Sachs (1989). A conclusion is presented in section VI.

II. History, Methodology and Overview

The first version of the model was a multi-region model based on the theoretical Mundell-Fleming-Dornbusch model with sticky prices. This was essentially a standard Keynesian model with the additional assumption of rational expectations in the asset markets. It formed the basis of the work in Sachs and McKibbin (1985), McKibbin and Sachs (1986a), McKibbin and Sachs (1986b), Ishii, McKibbin and Sachs (1986), Sachs (1986a) and the Brookings model comparison conference reported in Bryant et.al. (1988).

The new MSG2 model is based more firmly in micro-foundations. It relies heavily on the assumption that economic agents maximize intertemporal objective functions. This idea is very similar to the class of models known as Computable General Equilibrium (CGE) models[1] except that the concepts of time and dynamics are of fundamental importance in the MSG2 model. The various rigidities that are apparent in macroeconomic data are taken into account by allowing for deviations from the fully optimizing behavior. As with any modelling project that purports to describe reality, the tradeoff between theoretical rigor and empirical regularities is inevitable.

The MSG2 model can be described as a dynamic general equilibrium

---

[1] Such models are the basis of the work by Dixon et.al. (1982), Whalley (1985) and Deardoff and Stern (1986).

model of a multi-region world economy. In the present paper the regions modelled are the United States, Japan, Germany, the rest of the EMS (denoted REMS), the rest of the OECD economies (denoted ROECD), non-oil developing countries (LDCs), and OPEC. The model is of moderate size (about three dozen behavioral equations per industrial region). It is distinctive relative to most other global models in that it solves for a full intertemporal equilibrium in which agents have rational expectations of future variables. In theoretical conception, therefore, the model is close in design to intertemporal dynamic models of fiscal policy in Lipton and Sachs (1983) and Frenkel and Razin (1988). Those studies, like the present model, examine fiscal policy in an intertemporal perfect-foresight environment, with considerable attention given to intertemporal optimization and intertemporal budget constraints. Frenkel and Razin are noteworthy in being able to derive analytical results from their model, rather than relying on simulations, as in the current project.

The model has a mix of Keynesian and Classical properties by virtue of a maintained assumption of slow adjustment of nominal wages in the labor markets of the U.S., Germany, REMS and the ROECD (Japan is treated somewhat differently, as described below). Both the German and REMS regions are also assumed to experience long periods of "hysteresis" when unemployment emerges.

The model is solved in a linearized form, to facilitate policy optimization exercises with the model, and especially to use linear-quadratic dynamic game theory and dynamic programming solution techniques.[2] We have experimented with the full non-linear model and found that the properties of this model correspond closely to those of the linearized model, particularly over the initial years of any shocks. The global stability of the linearized model can be readily confirmed by an analysis of the model's eigenvalues.

In fitting the model to macroeconomic data we adopt a mix of standard CGE calibration techniques and econometric time series results. In CGE models, the parameters of production and consumption decisions are determined by assuming a particular functional form for utility functions and production functions and by assuming that the data from an expenditure share matrix or an input-output table represent an equilibrium of the model. For example, if utility is assumed to be a Cobb-Douglas nesting of the consumption of different goods, then the parameters of the utility function and therefore the demand functions for different goods are given by the expenditure shares found in the data. In this example, the demand function for each good in the system will have price and income elasticities of unity. In most cases the data will determine the parameters of the model although in some cases

[2] In general, quantity variables are linearized around their levels relative to potential GDP, while price variables are linearized in log form.

additional econometric analysis is required. The question of calibrating the model will be discussed further below.

The model has several attractive features which are worth highlighting. First, all stock-flow relationships are carefully observed. Budget deficits cumulate into stocks of public debt; current account deficits cumulate into net foreign investment positions; and physical investment cumulates into the capital stock. Underlying growth of Harrod-neutral productivity plus labor force growth is assumed to be 3 percent per region. Given the long-run properties of the model, the world economy settles down to the 3 percent steady-state growth path following any set of initial disturbances.

A second attractive feature is that the asset markets are efficient in the sense that asset prices are determined by a combination of intertemporal arbitrage conditions and rational expectations. By virtue of the rational expectations assumption and the partly forward-looking behavior of households and firms, the model can be used to examine the effects of anticipated future policy changes, such as the sequence of future budget deficit cuts called for by the Gramm-Rudman legislation in the U.S. Indeed, one of the difficulties of using the MSG2 model is that every simulation requires that the "entire" future sequence of anticipated policies be specified. In practice, forty year paths of policy variables, or endogenous policy rules, must be specified.

A third attractive feature of the model is the specification of the supply side. There are several noteworthy points here. First, factor input decisions are partly based on intertemporal profit maximization by firms. Labor and intermediate inputs are selected to maximize short-run profits given a stock of capital which is fixed within each period. The capital stock is adjusted according to a "Tobin's q" model of investment, derived along the lines in Hayashi (1979). Tobin's q is the shadow value of capital, and evolves according to a rational expectations forecast of future post-tax profitability.

Another point of interest regarding the supply side is the specification of the wage-price dynamics in each of the industrial regions. Extensive macroeconomic research has demonstrated important differences in the wage-price processes in the U.S., Europe, and Japan, and these differences are incorporated in the model. In particular, the U.S. and the ROECD (including Canada and Australia) are characterized by nominal wage rigidities arising from long-term nominal wage contracts. In Japan, on the contrary, nominal wages are assumed to be renegotiated on an annual, synchronized cycle, with nominal wages selected for the following year to clear the labor market on average. In the ROECD, nominal wages are assumed to be more forward looking than in the U.S., though real wages adjust slowly to clear the labor market. In Germany and the REMS we assume a degree of hysteresis. If unemployment rises it remains inertially above the market clearing level for a substantial period.

III The Model

The complete MSG2 model is presented in Appendix A. In this section the theoretical basis of these equations is outlined.

Each of the regions in the model produces a good which is an imperfect substitute in the production and spending decisions of the other regions. Each industrialized region produces one final good which is used for investment and consumption purposes in that region and in all of the other regions. The LDC and OPEC regions each produce one good which is a primary input in the production processes of the industrial regions. Demands for the outputs of the LDC and OPEC regions are therefore derived demands for the production inputs.

In the model version in this paper, only the five industrial country regions are fully modelled with an internal macroeconomic structure. In the LDC and OPEC regions, only the foreign trade and external financial aspects are modelled. Note that in referring to variables of the various regions, we will use the following notation: U.S. (U); Japan (J); Germany (G); Rest of the EMS (E); Rest of the OECD (R); OPEC (O); and LDC (L).

To understand the model, it is best to consider one bloc of the model, that of the U.S., and to indicate where necessary any differences in the modelling of the other OECD regions. Within each economy the decisions of households, firms and governments are modelled.

a Households

Households are assumed to consume a basket of goods in every period where the basket is made up of domestic goods (both public and private) and imported goods from each of the industrialized regions. They receive income to purchase the goods through providing labor services for production and receiving a return from holding financial assets.

Aggregate consumption (C) is nested in the following way:

$$C = C(C^d, C^m)$$

$$C^m = C^m(C^U_J, C^U_G, C^U_E, C^U_R)$$

where C^d is consumption of the domestic good, C^m is consumption of the imported bundle and C^U_i is consumption by the U.S. of goods produced in country i (i=J,G,E,R). Note that in the model listing the home country superscript is dropped for convenience.

The decision on how consumption expenditure is allocated between the various goods across time is based on a representative consumer who maximizes an intertemporal utility function[3] of the form:

$$\int_t^{\infty} [U(C_s)+V(G_s)]\ e^{-(\theta-n)s}\ ds$$

subject to the wealth constraint:

$$dF/ds = (r_s-n)F_s + W_sL_s(1-\tau_1)/P_s - P^c{}_sC_s/P_s \qquad (1)$$

$$F_s = M_s/P_s + B_s + q_sK_s + A_s + VOIL_s + VPE_s \qquad (2)$$

Utility in any period is written as an additively separable function of consumption of the private good (C) and the public good (G). In discounting the future stream of per capita consumption, the rate of time preference (θ) adjusted by the real growth rate (n) is used. The wealth accumulation equation given in (2) assumes that the change in real financial asset holdings (dF/ds) consists of a flow return on initial assets ((r-n)F), plus real after tax labor income less real expenditure on consumption. Financial assets are defined as real money balances (M/P), government bonds in the hands of the public (B), equity wealth (qK) and net foreign asset holdings (A). We also include in the definition of financial

[3] For notational convenience we will present all derivations assuming perfect foresight.

wealth, the value of claims to domestic oil reserves (VOIL)[4] and the present value of net profit arising from the pricing behavior of domestic firms in foreign markets (VPE)[5]. Note that P is the price of the domestic good and P^c is the price of the consumption good bundle ($p^c=P^c/P$). Note also that bonds are included as part of financial wealth but this does not imply that they are part of total wealth as the solution given below will show.

Setting up the Hamiltonian for this problem, assuming U(C)=logC, and solving gives the familiar first order conditions:

$$p^c_t\mu_t = 1/C_t \tag{3}$$

$$d\mu_t/dt = (\theta - r_t)\mu_t \tag{4}$$

where μ is the shadow value of consumption.
Solving these gives:

$$dp^cC/dt = (r_t - \theta)p^c_tC_t \tag{5}$$

This implies that if $r=\theta$, per capita real consumption is constant in the steady state.

---

[4] The treatment of oil is discussed in the next section.

[5] See section (f) below on the treatment of the pass-through of exchange rate changes into prices by firms operating in foreign markets.

The budget constraint given in equation (1) can be integrated and written as:

$$\int_{0}^{\infty} p^{c}{}_{s}C_{s} \; e^{-(Rs-n)s} \, ds - H_t = F_t \tag{6}$$

where H_t is real human wealth in period t and is defined:

$$\int_{t}^{\infty} W_s L_s (1-\tau_1)/P_s \; e^{-(Rs-n)s} \, ds = H_t \tag{7}$$

Real human wealth is the present discounted value of the entire future stream of real, after tax labor income.

where:

$$R_s = \int_{s}^{\infty} r_v \, dv$$

and r_i is the period i short-term real interest rate.

From the first order condition given in (5), we find:

$$\int_{0}^{\infty} p^{c}{}_{t}C_t \; e^{-(Rt-n)t} \, dt = p^{c}{}_{0}C_0/(\theta-n) \tag{8}$$

This can be substituted into (6) to give:

$$C_t = (\theta - n)\ \{F_t + H_t\}/p^c{}_t \tag{9}$$

and rewriting the human wealth condition gives:

$$dH_t/dt = (r_t - n)H_t - w_t L_t (1 - \tau_1) \tag{10}$$

This solution for aggregate consumption is a familiar life cycle model where, by the assumption of log utility, we find aggregate consumption is a linear function of real wealth which is comprised of financial wealth and human wealth. By assuming that aggregate consumption is a CES nesting of domestic and foreign goods we find equations for expenditure on each good as a function of aggregate expenditure.

$$C^d = [\beta_2{}^{\sigma_1}(P^c/P)^{\sigma_1}]C \qquad \sigma_1 = 1/(1-\beta_3) \tag{11}$$

$$C^m = [(1-\beta_2)^{\sigma_1}(P^c/P^m)^{\sigma_1}]C \tag{12}$$

where $\quad P^c C = P^m C^m + PC^d$

and $\quad P^{c(1-\sigma_1)} = \beta_2{}^{\sigma_1}\ P^{(1-\sigma_1)} + (1-\beta_2)^{\sigma_1}\ P^{m(1-\sigma_1)}.$

σ_1 is the elasticity of substitution between domestic and imported goods in the consumption bundle. Similarly, if the lower level nesting of imported goods is assume to be a CES function, we find further similar demand functions for each imported good. Note that with $\sigma 1=1$ this becomes the familiar linear expenditure system.

There is a large body of empirical evidence that suggests that aggregate consumption is partly determined along life-cycle lines, with considerable intertemporal consumption smoothing, and partly along simpler Keynesian lines (perhaps because of liquidity constrained households)[6]. Thus, we specify that consumption spending is a fixed proportion of current net-of-tax labor income (with no consumption smoothing of the labor income flow), as in standard Keynesian models, and a fixed proportion of wealth, as in standard life-cycle models with infinite-lived individuals. The aggregate consumption equation is in the form:

$$C = \beta_6(\theta - n)(F+H)P/P^c + (1-\beta_6)(Y-T) \qquad (\text{note } \theta=\beta_1) \tag{13}$$

We also introduce an additional term into the equation for human wealth. This is a risk premium that drives a wedge between the rate at which private individuals can borrow in the capital markets and the rate at which governments borrow.

These modifications to capture empirical regularities in aggregate consumption are assumed to not change the lower level demand functions. Note that in this model we assume $r>n$ which introduces another source of saddle point stability into the model. This assumption is necessary if human wealth is to be positive in the

---

[6] See for example Hayashi (1982) and Campbell and Mankiw (1987).

steady state.[7]

b. Firms

The cornerstone of aggregate supply in the model is a representative firm which maximizes its value by producing a single output Q at price P, subject to a two-input production function. All variables are written in terms of per efficiency labor units. Potential growth in the model is assumed to be 3 percent. Thus, aggregate production is given as:

$$Q = Q(V,N) \tag{14}$$

Gross output Q is a produced with value added V, and primary inputs N. In turn, V is produced with capital K and labor L, while N is produced with imports from the LDCs (N_L) and energy which consists of imports from OPEC (N_O) and domestic oil production (N_P):

$$V = V(K,L) \tag{15}$$

$$N = N(N_O, N_L, N_P) \tag{16}$$

We assume that domestic oil resources and imports of OPEC oil are perfect substitutes. Total oil demand as an intermediate input is

[7] Steady-state human wealth is $WL(1-\tau_1)/(r-n)$

assumed to be divided between the two sources based on historical shares. As already noted above, we also assume that households hold claims over domestic oil resources.

The capital stock changes according to the rate of fixed capital formation J and the rate of geometric depreciation δ:

$$dK/dt = J_t - (\delta+n)K_t \tag{17}$$

J is itself a composite good, produced with a Cobb-Douglas technology that has as inputs the domestic goods from the U.S. and the final goods of Germany, REMS, Japan and the ROECD. The price of J is simply a weighted sum of the prices of the home goods P (P^U for the U.S.) and the dollar import prices (E^iP^i, i = J, G, E, R) of goods from the other OECD regions :

$$J = \pi_i \ (Q^i)^{\beta_{18i}} \qquad i=\{U,J,G,E,R\}, \qquad \Sigma_i\beta_{18i}=1 \tag{18}$$

$$P^J = \pi_i \ (E^iP^i)^{\beta_{18i}} \qquad i=\{U,J,G,E,R\} \tag{19}$$

Following the cost of adjustment models of Lucas (1967) and Treadway (1969), it is assumed that the investment process is subject to rising marginal costs of installation, with total real investment expenditures I equal to the value of direct purchases of investment P^J*J/P, plus the per unit costs of installation. These per unit costs, in turn, are assumed to be a linear function of the rate of investment J/K, so that adjustment costs are $P^J*J\ [(\phi_o/2)(J/K)]/P$. Total investment expenditure is therefore:

$$I \quad = [\ P^J + P^J\ (\phi_o/2)\ \ (J/K)\,]\ J/P \qquad (20)$$

The goal of the firm is to choose inputs of L, N, and J to maximize intertemporal net-of-tax profits. In fact, the firm faces a stochastic problem, a point which is ignored in the derivation of the firm's behavior (in other word's, the firm is assumed to hold its estimates of future variables with subjective certainty). The firm's deterministic problem, formally stated, is:

Maximize:

$$\int_t^{\infty} [(1-\tau_2)(Q_s\ -\ (W_s/P_s)L_s\ -\ (P^N{}_s/P_s)N_s)\ -\ (P^J{}_s/P_s)I_s]\ e^{-(R_s-n)s}\ ds$$

subject to equations (14) through (20).

Solving the firms problem we find a set of conditions found in (21) to (24):

$$Q_L = w\ , \qquad (w=W/P) \qquad (21)$$

$$Q_N = p^n\ , \qquad (p^n=P^n/P) \qquad (22)$$

$$\lambda = p^J(1+\phi_o J/K)\ , \qquad (p^J=P^J/P) \qquad (23)$$

$$d\lambda_s/ds = (r+\delta)\lambda_s - (1-\tau_2)Q_K - .5p^I\phi_o(J/K)^2 \tag{24}$$

where λ is the shadow value of investment.

There are three key points from these solutions. First, inputs of L and N are hired to the point where the marginal productivity of these factors equal their factor prices. This gives equations for the derived demand for L and N given in (21) and (22).
The second point is seen by interpreting equation (24). Equation (24) can be integrated to find:

$$\lambda_t = \int_t^{\infty} [(1-\tau_2)Q_{Ks} + \Phi_K] \, e^{-(R_s+\delta)s} \, ds \tag{25}$$

Here Q_k is the marginal product of capital in the production function, and Φ_K ($=0.5p^I{}_s\phi_o[J_s/K_s]^2$) is the marginal product of capital in reducing adjustment costs in investment. λ is therefore the increment to the value of the firm from a unit increase in investment. It has a similar interpretation to Tobin's q. If we assume $q = P^J\lambda/P$, we can rewrite (23) as:

$$J = [(q-1)/\phi_o] \, K \tag{26}$$

The third point is that gross fixed capital formation can be written in terms of Tobin's "marginal" q as in (26).

In the specific application in the model, the gross output production function is taken to be a two-level Cobb-Douglas function in V and N, with V a Cobb-Douglas function of L and K, and N a Cobb-Douglas function of oil and non-oil primary inputs. Oil is then a Cobb-Douglas function of domestic production and imports from OPEC. Following the results in Hayashi (1979), the investment function derived in (26) is also modified, for empirical realism, by writing J as a function not only of q, but also of the level of flow capital income at time t. One argument for the inclusion of current profits is that it captures the existence of firms that are unable to borrow and lend as assumed by the theoretical derivation and therefore investment out of retained earnings. The modified investment equation is of the form:

$$J_t = \beta_{16} [(q-1)/\phi_o] K + (1-\beta_{16})[Q-(W/P)L-(P^N/P)N] \qquad (26')$$

The supply side of the U.S. block of the model is completed with the wage equation, which makes the nominal wage change a function of past consumer price changes ($\pi^c{}_{t-1}$), rationally expected future price changes ($\pi^c{}_t$), and the level of unemployment in the economy (labor demand ,L, relative to full employment, L^f), according to a standard Phillips curve mechanism:

$$d\log W/dt = \beta 22\ \pi^c{}_t + (1-\beta_{22})\pi^c{}_{t-1} + .2(l - l^f) \qquad (27)$$

where l^f represents the inelastically supplied full-employment stock of labor (in logs). The parameter β_{22} in (27) determines how much weight is given to backward-looking versus forward-looking price expectations.

As already noted, we allow for differences in the wage dynamics of the different regions. In the ROECD we also use equation (27). In Japan, we specify that wages are set one period ahead at their expected market clearing levels. Thus, let $({}_tW_{t+1})^f$ be the wage expected to clear the labor market at time t+1, in the sense that ${}_tL_{t+1} = L^f$. Then:

$$W^J_{t+1} = ({}_tW^J_{t+1})^f \tag{28}$$

Following Blanchard and Summers (1986) and Sachs (1986b), we build "hysteresis" into the labor markets in Germany and the REMS. For each of these regions we modify the wage equation in the following way:

$$d\log W/dt = \beta 22\ \pi^c{}_t + (1-\beta_{22})\pi^c{}_{t-1} + .1(L/L^* - 1) \tag{29}$$

$$L^* = L^f + 0.2(L^*_{t-1} - L^f) + 0.7(L_{t-1} - L^f) \tag{30}$$

In equation (29), wages respond to the difference between labor demand and the short-run natural rate (L^*). The short-run natural rate adjusts slowly to the long-run natural rate and it can deviate from the full employment level for a substantial period.

c. Government

We assume that the government in each country divides spending G among the final goods in the same proportion as does the private sector (this assumption is for convenience only), so that:

$$G^U_i \;/\; G^U \;=\; C^U_i \;/\; C^U \tag{31}$$

for $i = J, G, E, R$

The government finances this spending through company taxes, personal income taxes and issuing government debt. The government budget constraint can be written:

$$dB/dt = DEF = G - T + (r-n)B \tag{32}$$

Assuming a transversality condition that debt has value

i.e. $\lim_{s\to\infty} B_s e^{-(Rs-n)s} = 0$,

equation (32) can be integrated and written as:

$$B_t = \int_t^{\infty} (T_s - G_s)\, e^{-(Rs-n)s}\, ds$$

The current level of debt to gdp is the present value of future primary budget surpluses. With an outstanding stock of debt, if a government runs a budget deficit today it must run a budget surplus as some point in the future otherwise the debt will have no value.

In simulating fiscal policy we make several assumptions. The government can either choose policy exogenously or it choose it based on dynamic optimization of some objective function. In the case of an exogenous change in fiscal policy, it is important that tax and spending policies be consistent with the intertemporal budget constraint of the public sector. In particular, as already mentioned, starting from any initial stock of public debt, the discounted value of current and future taxes must equal the discounted value of government spending plus the initial value of outstanding public debt.

If the tax schedule were not subsequently altered, the stock of public debt would eventually rise without bound, at an explosive geometric rate. To prevent this, we assume that labor income taxes are increased each year by enough to cover the increasing interest costs on the rising stock of public debt. Letting B_0 be the pre-expansion stock of debt, the tax rule is therefore:

$$T_t=T_o+\tau_1(W/P\ L) + \tau_2\ [Q - (W/P)\ L - (P_N/P)N] + T_s \qquad (33)$$

Here, τ_1 is the average tax rate on labor income, and τ_2 is the average tax rate (corporate and personal) on capital income. T_s is a shift term in the tax schedule that rises along with the increase in interest payments on the public debt, $r_tB_t - r_0B_0$. It is assumed that T_s falls entirely on labor income (this assumption is made for convenience only, and will be modified in a later version of the model). T_0 is an exogenous tax shift parameter.

d. Financial Markets

Money is given a role is the model by notionally introducing it as a factor of production. It can be seen as a factor of production in the sense that the final produced good cannot be consumed until it is purchased with money. Using this derivation we can modify the producers decision by adding a first order condition similar to that for the other variable factors; the derived demand for money will be a function of output and the relative price of money. By specifying a CES technology in purchasing goods, we impose a unitary income elasticity but an interest elasticity proportional to the elasticity of substitution between money and the final good. The empirical money demand literature can be used to determine the interest elasticity and therefore an implied elasticity of substitution.

Asset markets are assumed to be perfectly integrated across the OECD regions. In the model calibrated on 1986 data which is basis of this paper we assume that capital controls in the REMS are not effective. Expected returns of loans denominated in the currencies of the various regions are equalized period to period, according to the following interest arbitrage relations:

$$i^i_t = i^j_t + ({}_tE^i_{jt+1} - E^i_{jt})/E^i_{jt} \tag{34}$$

Thus, we do not allow for risk premia on the assets of alternative currencies. We choose the assumption of perfect capital mobility and zero risk premia in light of the failure of the empirical exchange rate literature to demonstrate the existence of stable risk premia across international currencies.

e. Balance of Payments and the LDC and OPEC Blocs

Any trade imbalances are financed by flows of assets between countries. To determine net asset positions we make several simplifying assumptions. All new OPEC loans are assumed to be made proportionally to each region and new loans to the LDCs are also fixed in historical proportions. All other net flows are restricted to be consistent by imposing the constraint that current account balances and trade account balances sum to zero.
For the U.S., Japan, Germany, ROECD, and OPEC, the current account is determined under the assumption that domestic agents have free un-rationed access to international borrowing and lending at the international interest rate. As mentioned above, in the case of REMS, we no longer assume that capital flows from the REMS to other regions are inhibited. It is assumed for simplicity that all

international borrowing and lending takes place in dollar denominated assets. For the LDCs, in distinction, the scale of borrowing is set exogenously, under the assumption that the amount of loans available to the LDCs is rationed by country risk considerations.

For the goods of OPEC and the LDCs which feed into the production process of the industrialized regions, there is a single uniform world price of goods which applies in all markets at all times (i.e. the law of one price holds). Letting P^O be the dollar price of OPEC goods, we assume that P^O is a variable markup over a basket of OECD goods, so that:

$$P^O = P^O\ (P^U,\ E^J{}_U P^J,\ E^G{}_U P^G,\ E^E{}_U P^E,\ E^R{}_U P^R) * h\ (X^O) \tag{35}$$

with $h' > 0$.

Note that $E^i{}_U$ is in units of dollars per unit of currency i. The function P^O (.,.,.,.,.) is linear homogenous and increasing in the prices of the OECD goods. The function $h(X^O)$ makes the OPEC markup an increasing function of the total demand for OPEC exports X^O to the other regions. A similar equation governs the price of LDC commodities. The local currency price of OPEC goods in a non-U.S. region j is then given by $P^O{}_J = E^U{}_J * P^O$, according to the law of one price. A similar equation applies for the LDC commodity export.

f. The effect of Exchange Rates on Import Prices

Recent studies such as Baldwin and Krugman (1987) and Mann (1987) have pointed to the existence of a significant lag in the pass-through of exchange rate changes into import prices in the U.S. economy. The appreciation of the U.S. dollar during 1981-85 did not bring about an instantaneous and equivalent fall in import prices and the recent depreciation of the U.S. Dollar has not lead to a commensurate rise in import prices. To capture part of this effect we assume a lag in the exchange rate effect on import prices of the following form:

$$e^{i*}_t = e^{i*}_{t-1} + \beta_{23}\ (e^i_t - e^i_{t-1}) + (1-\beta_{23})(e^{*i}_{t-1} - e^{*i}_{t-2}) + .05(e^i_{t-1} - e^{*i}_{t-1}) \qquad (36)$$

where e^{i*}_t is log of the exchange rate that enters the pricing and demand equations in each country. This assumes that each foreign firm prices the same way in a particular country but possibly differently in different countries. For example this assumes that both Japanese and German firms selling goods in the U.S. market allow the same proportional flow on of exchange rate changes in pricing in the U.S. market. This behavior is consistent with a variety of arguments involving imperfect competition in international trade (see Dornbusch (1987) or Krugman (1986)). The profits and losses of the firms involved in exporting, is translated into the valuation of the firm in the original economies and therefore also into the wealth calculations for each economy.

g. Model Closure

The model is completed by assuming market clearing conditions. Prices in the U.S. (and the other OECD regions) are fully flexible within each period, so that demand for U.S. output (domestic demand plus export demand) equals output supply. Short term nominal interest rates adjust to clear the money market.

IV Calibration and Model Solution

a. Calibration

There are two issues which need to be dealt with in calibrating this model. The first is to choose behavioral parameters. The second is to choose a cross-section of data at some point in time, around which to linearize the model for game theoretic applications. The data and parameters must be internally consistent with the model specification. In finding parameters for the model we use a mix of techniques from the CGE literature as well as time series evidence.

In most CGE models both the data and the model parameters are manipulated to replicate an equilibrium of the model. In a dynamic model such as the MSG model, a corresponding procedure would be to choose a steady state of the model around which to calibrate. In principle, this is reasonable for a theoretical model because we could assume we start at a steady state since we were not concerned with recreating any actual year of data.[8] To replicate an actual data set is more problematic since we are trying to keep within the bounds consistent with this data set.

Our technique is to choose a set of behavioral parameters which fall within the range found in the many empirical studies of time series relations (e.g. factor shares and elasticities of substitution). Given this set of parameters and data for macro aggregates (e.g. output, consumption expenditure) which are based in part on data for 1986, we can use steady state relations in the model to generate other data (e.g. human wealth). A summary of the key features of this procedure follow. The notation used is that found in the model appendix. The actual values of parameters are listed in detail in Appendix A.

The real sector is calibrated using steady state and first order conditions where possible. We want to use as much actual data as possible to capture the relevance of the model especially since we linearize the model for the dynamic game applications. However, some steady-state conditions cannot be used. For example, the

---

[8] See McKibbin (1986) for this approach.

actual asset stocks for 1986 and actual trade flows in 1986 are not consistent with being in steady state; a positive holding of net foreign debt should be associated with a trade balance surplus. This aspect of the calibration is unavoidable. In terms of interpretation of the point around which the model is linearized, it can be interpreted as a point on the stable manifold of the model at which time the economy is adjusting towards to steady state.

For any equation in which adjustment occurs according to some share formulation, we assume the shares are 1986 shares. For example, the use of Japanese goods in U.S. investment is assumed to be equal to the ratio of Japanese goods in total U.S. consumption. Also the share of total LDC expenditure on each industrialized country's good is assumed equal to that in 1986. Any change in total LDC expenditure is then proportionately allocated between the goods from the different regions.

We select the growth rates (n) of each region at 3 percent per year and the real interest rate r0 at 5 percent. We also assume that the rate of time preference is equal to the real rate of interest. The choice of equal rates of time preference in each country is due to the problem that in infinite horizon multi-country models, one country would dominate the world eventually.[9] All initial prices are normalized at 1 (=0 in logs).

To ensure all equations are consistent we modify some data. Given the bilateral trade flows we have data on trade balances. Given values for Y,C,G we have by the goods market clearing identity to generate data for investment:

$I = Q - C - G - TB.$

Given assumed values for K and β_{15} (cost of adjusting capital) we can use the net investment equation:

$I = J\ (1+.5\beta_{15}J/K)$

to generate a value for J. We choose the positive valued solution. The next step is to find a value for q. We can use the equation for gross capital formation to find q:

$q = 1+(J/K)\beta_{15}$

The equation for the evolution of the shadow value of capital (q) can be solved to find the steady state marginal product of capital:

$dq/dK = [(r+\beta_{14})q \ - p^I(\beta_{15}/2)(J_t/K_t)^2\ ] \ / \ (1-\tau_2)$

We have that the share of capital in production, is a function of the marginal product of capital, the capital stock and output. Given the share of capital we have the share of labor. The real wage is normalized at unity and therefore we use the first order condition for labor demand to give labor measured in efficiency

---

[9] Alternative assumptions can be made such as allowing for unrelated agents in the model although this is somewhat artificial. See McKibbin (1986) for the application of this to a prototype model.

units. Given Y, K, L, factor shares and assumptions about factor substitutability, this implies a value for the constant (β_{10}) in the production function.

To calibrate the household sector of the model, we use similar techniques of appealing to first order conditions and steady state relations. The human wealth equation gives:

$$H = WL(1-\tau 1)/(r-n)$$

We can now generate a series for human wealth which, when combined with assumptions of initial asset holdings, gives a series for total wealth.

The assumptions about substitutability between consumption of different goods and initial shares in the utility function are based on empirical estimates of price elasticities. Consider equation (11) above, which can be rewritten in percentage change form:

$$c^d = c + \sigma_1 (p^c - p + \ln\beta_2)$$

where $p = \log P$

$c = \log C$

now dc^d/dp is the price elasticity of the demand for the domestic good (ϵ_{cp}). We find:

$$\epsilon_{cp} = -\sigma_1 .$$

In the case of the CES function there is a unique relation between the price elasticity and the elasticity of substitution in consumption. Taking the ratio of (11) and (12) it can be shown that:

$$C^d/C^m=[\beta_2 P^m/((1-\beta_2)P)]^{\sigma_1}$$

Solving for β_2, it can be shown that:

$$\beta_2 = (C^{ds_1})/(C^{ms_1}+C^{ds_1}) \qquad s_1 = 1/\sigma_1$$

We can now use these to find the shares (β_2) and elasticity of substitution (σ_1) given price elasticities and initial consumption levels or we can use any empirical evidence on the elasticity of substitution and expenditure shares to find the implied price elasticities.

b. Model Solution

Solving a model such as the MSG model which assumes rational expectations in different markets is not a straightforward exercise. Forward looking variables such as asset prices, consumption and investment decisions are conditioned on the entire future path of all variables in the model. We are presented with a two-point, boundary value problem; values for inherited variables (state variables) are known and the expected path of exogenous variables are assumed to be known but for forward looking variables we can only assume some terminal conditions. Various techniques are available for solving these models such as the Multiple

Shooting algorithm outlined in Lipton et.al. (1982) and the technique in Fair and Taylor (1983) for non-linear models. An analytical solution is provided by Blanchard and Kahn (1980) for linear models. We use a technique which we will call the MSG technique. We will only briefly introduce it here but refer the reader to McKibbin (1987) for more details and a comparison of this technique with the other techniques.

The MSG technique is based on a backward recursion algorithm used for solving dynamic games. An advantage of this technique is that we can solve for dynamic game equilibria as well as the standard rational expectation equilibria with minimal computational cost.

The model is first linearized. This is done because we use the model for dynamic games which require linearity for a unique solution. [10] Once linearized we express the model in minimal state-space representation. Because the model has been linearized, we know from the Blanchard-Kahn technique that we can express the jumping (or expectation) variables as a function of the known state variables in any period and the future path of exogenous variables. The goal of our technique is to find this rule numerically. In the algorithm used, we first assume a terminal period (T) in which we

---

[10] An earlier non-linear version of the model was solved using the Fair-Taylor technique and it was found to have very similar properties to the linear model. Given the potential saving in computing time and computing constraints we continue to use the linear model.

impose stationarity conditions on the expected variables in the model. The model is then solved in period T-1, conditional on initial conditions in period T-1, as well as conditional on the terminal solution we have imposed for period T. The period of solution is then moved back to period T-2 and the model is solved again conditional on the path for expected variables we found for the period T-1 solution and the imposed terminal solution. Each period we find a rule linking jumping variables to state variables and exogenous variables which is, in general, time dependent. We continue the moving backwards and solving forwards until the rule converges to a time invariant rule. The rule itself is independent of the shock or the initial conditions.

By compressing the entire future of the economy into a rule of this form, we have transformed the model into a standard difference equation model. For any shock, the rule we have found will give the value of the jumping variables. The model can then be simply solved forward. The extension of this technique to dynamic games will not be discussed but is available in McKibbin (1987).

In summary, each time a new model is used, the search for saddle point stable rules for the jumping variables need only be performed once. After the rule is found, any shock to exogenous variables or initial conditions can be simply solved as with any standard difference equation model.

VI Conclusion

This paper has presented the theoretical foundations of the MSG2 model. The model has its roots in both the CGE modelling literature and the recent theoretical macroeconomics literature. The importance of forward-looking expectations in consumption decisions, investment decisions, and directly in financial markets as well as the role of intertemporal budget constraints distinguishes the model from other global empirical models. Simulation results in McKibbin and Sachs (1989), especially those that highlight the role of expectations about future policy, indicate the potential usefulness of an approach such as the MSG model. This paper has not attempted to undertake any model validation exercises. The tracking performance of the model over the period from 1979 to 1988 is available in McKibbin (1989).

References

Baldwin R. and P. Krugman (1987) "The persistence of the U.S. Trade Deficit". Brookings Papers on Economic Activity, 1:87, pp1-43.

Blanchard O. and C. Kahn (1980) "The Solution of Linear Difference Models Under Rational Expectations", Econometrica 48, pp.305-311.

Blanchard O. and L. Summers (1986) "Hysteresis and the European Unemployment Problem." NBER Macroeconomics Annual, (1).

Bruno M. and J. Sachs (1985) Economics of Worldwide Stagflation. Harvard University Press.

Bryant, R., Henderson D., Holtham, G., Hooper, P. and S. Symansky (eds) (1988) Empirical Macroeconomics for Open Economies. Brookings Institution, Washington, D.C.

Campbell J. And N.G. Mankiw (1987) "Permanent Income, Current Income and Consumption" NBER Working Paper 2436.

Dixon P.B., Parmenter, B.R., Sutton, J and D. Vincent (1982) ORANI: A Multisectoral Model of the Australian Economy, North Holland.

Deardorff A.V. and R.M. Stern (1986) The Michigan Model of World Production and Trade. MIT Press.

Dornbusch R. (1976) "Expectations and Exchange Rate Dynamics" Journal of Political Economy 84, pp 1161-76.

Dornbusch R. (1987) "Exchange Rates and Prices", American Economic Review, 77, pp93-106.

Fair, R. and J. Taylor (1983) "Solution and Maximum Likelihood Estimation of Dynamic Non-Linear Rational Expectations Models" Econometrica 51(4), pp. 139-178.

Frenkel, J. and A. Razin (1988) Fiscal Policies in the World Economy, MIT Press.

Hayashi, F. (1979) "Tobins Marginal q and Average q: A Neoclassical Interpretation." Econometrica 50, pp.213-224.

Hayashi, F. (1982) "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables. Journal of Political Economy 90(4) pp 895-916.

Ishii, N. McKibbin, W.J. and J. Sachs (1985) "The Economic Policy Mix, Policy Cooperation and Protectionism: Some Aspects of Macroeconomic Interdependence Among the United States, Japan and Other OECD Countries." Journal of Policy Modelling 7(4), pp.533-572.

Krugman P. (1986) "Pricing to Market when Exchange Rates Change", National Bureau of Economic Research Working Paper 1927.

Lipton D., Poterba J., Sachs J. and L. Summers (1982) "Multiple Shooting in Rational Expectations Models". Econometrica 50 , pp1329-1333.

Lipton, D. and J. Sachs (1983) "Accumulation and Growth in a Two-Country Model," Journal of International Economics 15, pp.135-159.

Lucas, R. E.(1967) "Adjustment Costs and the Theory of Supply." Journal of Political Economy 75(4), pp.321-334

Mann C. (1987) "After the Fall: The Declining Dollar and Import Prices" Federal Reserve Board.

McKibbin, W.J. (1986) "The International Coordination of Macro-economic Policies." Ph.D. Dissertation, Harvard University.

McKibbin, W.J. (1987) "Numerical Solution of Rational Expectations Models, With and Without Strategic Behaviour" Reserve Bank of Australia, Research Discussion Paper 8706.

McKibbin, W.J. (1989) "The World Economy from 1979 to 1988: Results from the MSG2 Model" Brookings Discussion Paper in International Economics no. 72.

McKibbin, W.J. and J. Sachs (1986a) "Coordination of Monetary and Fiscal Policies in the OECD." National Bureau of Economic Research Working Paper No.1800, in Jacob Frenkel (Ed) International Aspects of Fiscal Policy. University of Chicago Press.

McKibbin, W.J. and J. Sachs (1986b) "Comparing the Global Performance of Alternative Exchange Arrangements." National Bureau of Economic Research Working Paper No.2000, revised in Journal of International Money and Finance,7, pp 387-410.

McKibbin, W.J. and J. Sachs (1989) "Implications of Policy Rules for the World Economy", NBER Working Paper and forthcoming in Bryant et al. (1989) Macroeconomic Policies in an Interdependent World, IMF/Brookings/CEPR.

Masson P., S. Symansky, R. Haas and M. Dooley (1988) "Multimod - A Multi-region Econometric Model" Part II, IMF Staff Studies for the World Economic Outlook, July.

Poterba, J. and L. Summers (1987) "Recent U.S. Evidence on Budget Deficits and National Savings," NBER Working Paper No.2144, February.

Sachs, J. (1982) "Energy and Growth Under Flexible Exchange Rates: A Simulation Study." in Bhandari, J. and B. Putnam (Eds.) The International Transmission of Economic Disturbances Under Flexible Rates. MIT Press, Cambridge, MA.

Sachs J. (1986a) "The Dollar and The Policy Mix" Brookings Papers on Economic Activity

Sachs, J. (1986b) "High Unemployment in Europe: Diagnosis and Policy Implications." National Bureau of Economic Research Working Paper 1850.

Sachs, J. and W. J. McKibbin (1985) "Macroeconomic Policies in the OECD and LDC External Adjustment." National Bureau of Economic Research Working Paper 1534.

Taylor, J. (1980) "Aggregate Dynamics and Staggered Contracts." Journal of Political Economy 88, pp.1-23.

Taylor J. (1988) "The Treatment of Expectations in large Multi-Country Econometric Models" in Bryant et.al. (1988).

Treadway, A. (1969) "On Rational Entrepreneurial Behavior and the Demand for Investment." Review of Economic Studies 36(106), pp.227-239.

Whalley J. (1985) Trade Liberalization Among the Major World Trading Areas, MIT Press.

APPENDIX A: THE MSG2 MODEL OF THE WORLD ECONOMY

Equation Listing

This model appendix lists the equations for the MSG model. The structure of each industrialized economy is similar and therefore only the U.S. equations are listed except where major differences between countries exist. Note that each variable is time dimensioned and country dimensioned. Where possible these two dimensions will be dropped for expositional reasons.

e.g. C^j_t (consumption of country j in period t) is written C;

$C_i{}^j{}_t$ (consumption by country j of goods from country i in period t) is written C_i;

A subscript/superscipt i refers to the set i={J,G,E,R} except where noted

A subscript/superscipt k refers to the set k={L,O}

Country Neumonics:

(U) United States;

(J) Japan;

(G) Germany;

(E) Rest of the EMS;

(R) Rest of the OECD;

(L) Developing Countries

(O) OPEC.

HOUSEHOLDS

(i) **Utility**

$$U_t = \int_t^{\infty} \log C_s \, e^{-(\beta 1-n)s} \, ds$$

$$C = [\beta_2 (C^d)^{\beta_3} + (1-\beta_2)(C^m)^{\beta_3}]^{(1/\beta_3)}$$

$$C^m = [\Sigma_i \ \beta_{5i} (C_i)^{\beta_4}]^{(1/\beta_4)} \qquad \Sigma_i \ \beta_{5i} = 1$$

(ii) **Demand Functions**

$$C = \beta_6(\beta_1 - n)(F+H)P/P^c + (1-\beta_6)[WL(1-\tau_1)/P+(r-n)F-TAX]P/P^c$$

$$C^d = [\beta_2^{\sigma_1}(P^c/P)^{\sigma_1}](C+G) \qquad \sigma_1 = 1/(1-\beta_3)$$

$$C^m = [(1-\beta_2)^{\sigma_1}(P^c/P^m)^{\sigma_1}](C+G)$$

$$C_i = [\beta_{5i}^{\sigma_2}(P^m/E^{i*}P^i)^{\sigma_2}]C^m \qquad \sigma_2 = 1/(1-\beta_4)$$

$$P^{c(1-\sigma_1)} = \beta_2^{\sigma_1} \ P^{(1-\sigma_1)} + (1-\beta_2)^{\sigma_1} \ P^{m(1-\sigma_1)}$$

$$P^{m(1-\sigma_2)} = \Sigma_i \ \beta_{5i}^{\sigma_2}(E^{i*}P^i)^{(1-\sigma_2)}$$

$${}_tH_{t+1} = (1+\beta_7+r-n)H_t - W(1-\tau_1)L/P + TAX$$

FIRMS

(i) **Production Function**

$$Q = \beta_{10}(V)^{\beta_{11}} (N)^{(1-\beta_{11})}$$

$$V = (K)^{\beta_{12}} (L)^{(1-\beta_{12})}$$

$$N = \beta_{13} \prod_k (N_k)^{\beta_{14k}}$$

(ii) **Factor demands**

$$L = \beta_{11}(1-\beta_{12})(PQ/W)$$

$$N = (1-\beta_{11})(PQ/P^n)$$

$$N_L = \beta_{14L} P^n N/(E^L P^L)$$

$$N_O = \beta_{24}\beta_{14O} P^n N/(E^O P^O)$$

$$N_P = (1-\beta_{24})\beta_{14O} P^n N/(E^O P^O)$$

$$K_{t+1} = (1-\beta_{15}-n)K_t + J_t$$

$$I = [P^J + P^J (\beta_{17}/2) (J/K)] J/P$$

$$J = \beta_{16}[(q-1)/ \beta_{17}] K + (1-\beta_{16})[Q-(W/P)L-(P^n/P)N]$$

$$J_i = \beta_{18i} J/\Lambda_i \qquad \Sigma\beta_{18i}=1$$

$$_tq_{t+1} = (1+r+\beta_{15}+\beta_{19})q_t - (1-\tau_2)\beta_{11}\beta_{12}(Q/K) - (P^J/P)(.5\beta_{17})(J/K)^2$$

$$P^n = \Lambda_k \ (E^kP^k)^{\beta_{14k}}$$

$$P^J = \Lambda_i \ (E^iP^i)^{\beta_{18i}}$$

$$_tVOIL_{t+1} = (1+\beta_7+r-n)VOIL_t - N_P P^O E^O/P$$

$$_tVPE_{t+1} = (1+\beta_7+r-n)VPE_t - \Sigma_i \ (C^i + I^i)(E^i/E^{i*})$$

ASSET MARKETS

$$M/P = Q^{\beta_{20}} \ i^{\beta_{21}}$$

$$F = B + M/P + qK + A + VPE + VOIL$$

$$A = A_L{}^U - \Sigma_i \ A_U{}^i \qquad i=\{J,G,E,R,P\}$$

$$\Lambda^i = P^iE^i/P$$

$$\Lambda^{i*} = P^iE^{*i}/P$$

$$\Lambda^k = P^kE^k/P$$

$$i_t = r_t + {}_t\pi_t$$

$$r_t = R_t - ({}_tR_{t+1} - R_t)/R_t$$

$$r_t = r^i_t + ({}_t\Lambda^i_{t+1} - \Lambda^i_t)/\Lambda^i_t$$

$${}_t\pi_t = ({}_tP_{t+1} - P_t)/P_t$$

$${}_t\pi^c_t = ({}_tP^c_{t+1} - P^c_t)/P^c_t$$

$$E^{i*}_t/E^{i*}_{t-1} = (E^i_t/E^i_{t-1})^{\beta 23}\ (E^{*i}_{t-1}/E^{*i}_{t-2})^{(1-\beta 23)}(E^i_{t-1}/E^{*i}_{t-1})^{.05}$$

GOVERNMENT SECTOR

$$DEF = g + rB - T$$

$$T = TAX + \tau_1(W/P)L + \tau_2[Q-(W/P)L-(P^n/P)N]$$

$$TAX = rB + TAXE$$

$$B_{t+1} = (1-n)B_t + DEF_t$$

WAGE SETTING

U.S., ROECD:

$$w_t = (W_{t+1}-W_t)/W_t$$

$$w_t = \beta_{22}\ {}_t\pi^c{}_t + (1-\beta_{22})\pi^c{}_{t-1} + .2(L/L^f-1)$$

Japan:

$$w^J{}_{t+1} = ({}_tw^J{}_{t+1})^f$$

Germany, REMS :

$$w_t = \beta_{22}\ {}_t\pi^c{}_t + (1-\beta_{22})\pi^c{}_{t-1} + .2(L/L^*-1)$$

$$L^* = L^f + 0.2(L^*{}_{t-1}-L^f) + 0.7(L_{t-1}-L^f)$$

BALANCE OF PAYMENTS

$$X = \Sigma_i\ (C^i + I^i)$$

$$IM = \Sigma_i\ \Lambda^{i*}(C_i + I_i) + \Sigma_k\ \Lambda^k N_k$$

$$TB = EX - IM$$

$$CA = TB + rA$$

$$A_{t+1} = (1-n)A_t + CA_t$$

MARKET EQUILIBRIUM

$$Q = (P^c/P)(C+G) + (P^J/P)I + TB + (P^n/P)N$$

$$M = M^s$$

<u>LDC Equations</u>

note $i=\{U,J,G,E,R,O\}$
$j=\{U,J,G,E,R\}$

$$P^L = \Lambda_i (E^i P^i)^{\mu_{1i}} \; (X^L)^{\mu_2} \qquad \Sigma\mu 1i=1$$

$$X^L = \Sigma_j \; N_L^j \; + \; C_L^O$$

$$IM^L = \Sigma_i \; C_i^L \Lambda^i$$

$$C_i^L \Lambda^i = \mu_{1i}(IM^L)$$

$$TB^L = X^L - IM^L$$

$$CAL = CAL_0$$

$$DEBT = \Sigma_i \; A_L^i \Lambda^i$$

$$DEBT_{t+1} = DEBT_t (1-n) - CA^L$$

$$A_L^i{}_{t+1} \Lambda^i{}_t = (1-n) A_L^i{}_t \Lambda^i{}_t + \mu_{3i}(DEBT_{t+1} - (1-n) DEBT_t)$$

OPEC Equations

note $i=\{U,J,G,E,R,L\}$
$j=\{U,J,G,E,R\}$

$$P^O = \Lambda_i (E^i P^i)^{\phi_{1i}} (X^O)^{\phi_2} \qquad \Sigma\phi 1i=1$$

$$X^O = \Sigma_j N_O^j + C_O^L$$

$$IM^O = \Sigma_i C_i^O \Lambda^i$$

$$C_i^O \Lambda^i = \phi_{1i}(IM^O)$$

$$TB^O = X^O - IM^O$$

$$CA^P = \phi_4 0.29 X^P \Lambda^P - (0.29-n)A^P$$

$$A^P = \Sigma_i A_i^P$$

$$A^P_{t+1} = (1-n)A^P_t + CA^P_t$$

$$A_i{}^P{}_{t+1} = (1-n)A_i{}^P{}_t + \phi_{3i}(A^P{}_{t+1} - (1-n)A^P{}_t)$$

Variable Definitions

A_i^j real claims by country j against country i

A^i total real claims held by country i against other countries

B real government debt

C real consumption of total bundle of goods

C^d real consumption of domestic bundle of goods

C^m real consumption of imported bundle of goods

C_i^j consumption by country i of country j good

CA real current account balance

DEBT LDC debt

DEF real budget deficit

E^i_j nominal exchange rate (units of currency j per unit of currency i; e.g. E^j_u is dollars per yen)

E^{*i}_j nominal exchange rate that enters the price of home country exports in foreign markets

F real financial wealth

G real government expenditure on goods

H real human wealth

i short nominal interest rate

I nominal investment expenditure inclusive of adjustment costs

J gross fixed capital formation

J_i^j imports by country j of investment goods from country i

K capital stock

L demand for labor

L^f full employment labour demand

M nominal money supply

N basket of intermediate inputs used in production

N_i^j import by country j of intermediate inputs from country i

N_P domestic production of oil

n growth rate of population plus labor-augmenting technical change

P price of domestic goods

P^m price of basket of imported goods

P^c price of a basket of imported and domestic goods

P^I price of basket of investment goods

P^n price of basket of intermediate goods

π product price inflation

π^c consumer price inflation

Q real gross output

q Tobin's q

R long real interest rate

r short real interest rate

T total nominal tax receipts

TAX lump sum tax on households

TAXE exogenous tax

TB trade balance in real domestic good units

V Intermediate good produced with domestic factors

VOIL Value of future stream of domestic oil production

VPE Value of net profit from slow pass through of exchange rate changes into foreign prices of export goods

W nominal wage

w rate of change of nominal wage

X real exports in domestic good units

IM real imports in domestic good units

τ_1 tax rate on household income

τ_2 tax rate on corporate profits

σ_1 elasticity of substitution between domestic and imported goods

σ_3 elasticity of substitution betweem capital and labor

Λ^i_j relative price of country i to j good (real exchange rate)

Λ^{*i}_j relative price of country i to j good (real exchange rate) adjusted for short term pricing behavior in foreign markets

Parameter Values

US

$\beta1 = 0.050$	$\beta2 = 0.932$	$\beta3 = 0.000$
$\beta4 = 0.000$	$\beta5j = 0.344$	$\beta5g = 0.106$
$\beta5r = 0.420$	$\beta5e = 0.130$	$\beta6 = 0.300$
$\beta7 = 0.100$	$\beta8 = 0.000$	$\beta10 = 1.000$
$\beta11 = 0.937$	$\beta12 = 0.350$	$\beta13 = 1.000$
$\beta14l = 0.364$	$\beta14o = 0.636$	$\beta15 = 0.100$
$\beta16 = 0.300$	$\beta17 = 20.000$	$\beta18u = 0.902$
$\beta18j = 0.034$	$\beta18g = 0.010$	$\beta18e = 0.013$
$\beta18r = 0.041$	$\beta19 = 0.060$	$\beta20 = 1.000$
$\beta21 = -0.600$	$\beta22 = 0.400$	$\beta23 = 0.500$
$\beta24 = 0.118$	$\tau1 = 0.350$	$\tau2 = 0.300$

Japan

$\beta1 = 0.050$	$\beta2 = 0.958$	$\beta3 = 0.000$
$\beta4 = 0.000$	$\beta5u = 0.502$	$\beta5g = 0.075$
$\beta5r = 0.327$	$\beta5e = 0.097$	$\beta6 = 0.300$
$\beta7 = 0.100$	$\beta8 = 0.000$	$\beta10 = 1.000$
$\beta11 = 0.952$	$\beta12 = 0.350$	$\beta13 = 1.000$
$\beta14l = 0.370$	$\beta14o = 0.630$	$\beta15 = 0.100$
$\beta16 = 0.300$	$\beta17 = 20.000$	$\beta18j = 0.944$
$\beta18u = 0.028$	$\beta18g = 0.004$	$\beta18e = 0.005$
$\beta18r = 0.018$	$\beta19 = 0.060$	$\beta20 = 1.000$
$\beta21 = -0.600$	$\beta22 = 0.200$	$\beta23 = 0.750$
$\beta24 = 0.434$	$\tau1 = 0.350$	$\tau2 = 0.300$

Germany

$\beta1 = 0.050$	$\beta2 = 0.783$	$\beta3 = 0.000$
$\beta4 = 0.000$	$\beta5u = 0.079$	$\beta5j = 0.075$
$\beta5r = 0.302$	$\beta5e = 0.545$	$\beta6 = 0.300$
$\beta7 = 0.100$	$\beta8 = 0.000$	$\beta10 = 1.000$
$\beta11 = 0.947$	$\beta12 = 0.350$	$\beta13 = 1.000$
$\beta14l = 0.620$	$\beta14o = 0.380$	$\beta15 = 0.100$
$\beta16 = 0.300$	$\beta17 = 20.000$	$\beta18g = 0.615$
$\beta18u = 0.116$	$\beta18j = 0.029$	$\beta18e = 0.210$
$\beta18r = 0.030$	$\beta19 = 0.060$	$\beta20 = 1.000$
$\beta21 = -0.600$	$\beta22 = 0.300$	$\beta23 = 0.750$
$\beta24 = 0.334$	$\tau1 = 0.350$	$\tau2 = 0.300$

REMS

$\beta 1 = 0.050$	$\beta 2 = 0.743$	$\beta 3 = 0.000$
$\beta 4 = 0.000$	$\beta 5u = 0.085$	$\beta 5g = 0.278$
$\beta 5j = 0.038$	$\beta 5r = 0.599$	$\beta 6 = 0.300$
$\beta 7 = 0.100$	$\beta 8 = 0.000$	$\beta 10 = 1.000$
$\beta 11 = 0.940$	$\beta 12 = 0.350$	$\beta 13 = 1.000$
$\beta 14l = 0.498$	$\beta 14o = 0.502$	$\beta 15 = 0.100$
$\beta 16 = 0.300$	$\beta 17 = 20.000$	$\beta 18e = 0.637$
$\beta 18u = 0.218$	$\beta 18g = 0.101$	$\beta 18j = 0.014$
$\beta 18r = 0.031$	$\beta 19 = 0.060$	$\beta 20 = 1.000$
$\beta 21 = -0.600$	$\beta 22 = 0.300$	$\beta 23 = 0.750$
$\beta 24 = 0.464$	$\tau 1 = 0.350$	$\tau 2 = 0.300$

ROECD

$\beta 1 = 0.050$	$\beta 2 = 0.771$	$\beta 3 = 0.000$
$\beta 4 = 0.000$	$\beta 5u = 0.209$	$\beta 5g = 0.183$
$\beta 5j = 0.073$	$\beta 5e = 0.536$	$\beta 6 = 1.000$
$\beta 7 = 0.100$	$\beta 8 = 0.000$	$\beta 10 = 1.000$
$\beta 11 = 0.937$	$\beta 12 = 0.350$	$\beta 13 = 1.000$
$\beta 14l = 0.364$	$\beta 14o = 0.636$	$\beta 15 = 0.100$
$\beta 16 = 0.300$	$\beta 17 = 20.000$	$\beta 18r = 0.551$
$\beta 18j = 0.033$	$\beta 18g = 0.082$	$\beta 18e = 0.241$
$\beta 18u = 0.094$	$\beta 19 = 0.060$	$\beta 20 = 1.000$
$\beta 21 = -0.600$	$\beta 22 = 0.400$	$\beta 23 = 0.750$
$\beta 24 = 0.132$	$\tau 1 = 0.350$	$\tau 2 = 0.300$

LDC

$\mu 1u = 0.219$	$\mu 1j = 0.198$	$\mu 1g = 0.116$
$\mu 1e = 0.190$	$\mu 1r = 0.178$	$\mu 1o = 0.099$
$\mu 2 = 0.500$	$\mu 3u = 0.364$	$\mu 3j = 0.129$
$\mu 3g = 0.124$	$\mu 3e = 0.134$	$\mu 3r = 0.167$
$\mu 3o = 0.081$		

OPEC

$\phi 1u = 0.129$	$\phi 1j = 0.145$	$\phi 1g = 0.102$
$\phi 1e = 0.211$	$\phi 1r = 0.193$	$\phi 1o = 0.220$
$\phi 2 = 0.500$	$\phi 3u = 0.714$	$\phi 3j = 0.071$
$\phi 3g = 0.071$	$\phi 3e = 0.071$	$\phi 3r = 0.071$
$\phi 3o = 0.000$	$\phi 4 = 3.127$	